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THE INFLUENCE OF DELAYS IN ELASTICITY AND DAMPING ON AUTO-PARAMETRIC OSCILLATIONS

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Abstract. The interaction of self-oscillations and parametric oscillations under the influence of delayed elastic and damping forces in the presence of a limited power energy source in the system is considered. To construct solutions to a nonlinear system of equations, the direct linearization method was used, which contains the linearization accuracy parameter. Using this method, equations of non-stationary and stationary motions are obtained to determine the amplitude, phase of oscillations and the speed of the energy source. Based on the Routh-Hurwitz criteria, the stability of stationary modes of motion is considered and the stability conditions are derived. In order to obtain information about the effect of elastic delays and damping on the dynamics of the system, calculations were performed for various combinations of their values. The corresponding amplitude-frequency characteristics and a graph of the load on the energy source from the side of the oscillatory system are constructed.

Keywords: *self-oscillations, parametric oscillations, energy source, limited power, delay, elasticity, damping, method, linearization.*

Introduction. In many problems [1-9, etc.], it becomes necessary to take into account the phenomenon of hysteresis (delay). Among them, hysteresis can be noted: plastic in mechanics, ferromagnetic and dielectric in physics, in control problems, in biology, in automatic control, etc. The carriers of hysteresis are part of a more complex system and therefore should be considered together. At cyclically changing stresses, the maximum amplitude of which is significantly lower than the elastic limit, dynamic hysteresis is observed, which is caused by inelasticity or viscoelasticity. "In case of inelasticity, in addition to purely elastic deformation (corresponding to Hooke's law), there is a component that completely disappears when stress is removed, but with some delay, and with viscoelasticity, this component does not completely disappear with time" [Wikipedia]. The retardation effects appear depending on the yield and discontinuous deformation, specimen geometry, loading conditions and modes, as well as on the properties of the loading system. Hysteresis also occurs as a result of thermoelasticity, magnetoelastic phenomena, changes in the position of point defects, etc. The delay has a significant effect on the control process and the stability of the system, it can lead to oscillations in it, which arise, for example, in servo systems, regulators, rolling mills, etc. Therefore, the study of the effects caused by the influence of the delay is of great practical interest.

For the analysis of nonlinear systems with delay, approximate methods are used (Bogolyubov-Mitropol'skiy averaging, harmonic linearization, energy balance, etc.

[10-19]), which are characterized by significant labor and time costs depending on the type of nonlinearity characteristics. Large expenditures of labor and time are one of the main problems in the analysis of the dynamics of nonlinear systems. With reference to works [20-23], this is indicated in [24], where it is noted that this problem exists for the study of coupled oscillatory networks that play an important role in chemistry, biology, physics, electronics, neural networks, etc. The method of direct linearization (MDL), described in [25-31, etc.], is fundamentally different from these methods. The advantages of the MDL over the well-known methods of analyzing nonlinear systems are simplicity and the associated low (several orders of magnitude less) labor and time costs, the absence of laborious and complex approximations of various orders, the possibility of obtaining final design relations regardless of the specific type and degree of nonlinearity. Using the MDL, below we consider the effect of delays in elasticity and damping on mixed parametric and self-oscillations in the presence of a limited power source in the system that supports the functioning of the system.

Model. Consider the well-known model (Fig.1) of a mechanical frictional self-oscillating system [32-35]. Nonlinear differential equations describing its motion, taking into account the delays in elasticity and damping, have the form

$$m\ddot{x} + k_0\dot{x} + c_0 x = T(U) - bx\cos\nu t - k_\eta \dot{x}_\eta - c_\tau x_\tau \qquad (1)$$
$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T(U)$$

where $k_0 = const$, $c_0 = const$, $T(U)$ is a nonlinear friction force that depends on the relative velocity $U = V - \dot{x}$ and causes self-oscillations, $V = r_0\dot{\varphi}$, $\dot{x}_\eta = \dot{x}(t-\eta)$, $x_\tau = x(t-\tau)$, $\eta = const$, $\tau = const$, η and τ are delays, $r_0 = const$ is the radius of the point of application of the friction force $T(U)$, I is the total moment of inertia rotating parts, $M(\dot{\varphi})$ is the difference between the torque of the power source and the torque of the forces of resistance to rotation, $\dot{\varphi}$ is the speed of rotation of the engine.

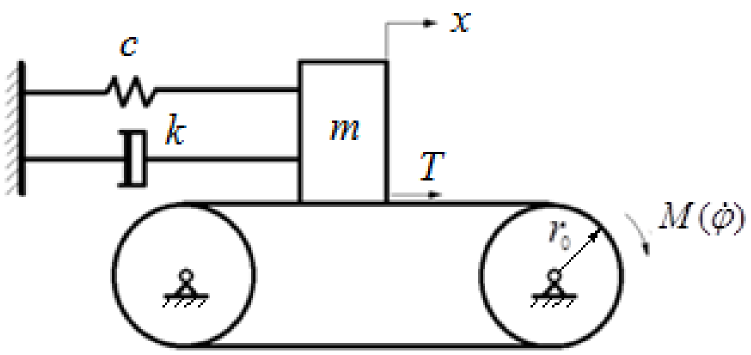

Fig.1. System model.

Let us imagine the friction force $T(U)$ with a widely used in practice falling characteristic of the coefficient of friction from the sliding speed, which was also observed when considering the problem of measuring friction forces in space conditions [36]:

$$T(U) = T_0\left[\operatorname{sgn} U + f(\dot{x})\right], \qquad f(\dot{x}) = \sum_{n=0}^{3} \delta_n \dot{x}^n \tag{2}$$

$$\operatorname{sgn} U = \begin{cases} 1 & \text{at} \quad U > 0 \\ -1 & \text{at} \quad U < 0 \end{cases}$$

Here T_0 is the normal reaction force, $T_0 \le T(0) \le T_0$, $\delta_0 = -\alpha_1 V + \alpha_3 V^3$, $\delta_1 = \alpha_1 - 3\alpha_3 V^2$, $\delta_2 = 3\alpha_3 V$, $\delta_3 = -\alpha_3$, α_1 and α_3 are constants.

Using the MDL [25-31], replace the nonlinear function $f(\dot{x})$ with a linear

$$f_*(\dot{x}) = B_T + k_T \dot{x} \tag{3}$$

where B_T and k_T are the linearization coefficients.

The B_T and k_T coefficients are determined by the expressions

$$B_T = \delta_0 + N_2 \delta_2 \upsilon^2, \qquad k_T = \delta_1 + \delta_3 \bar{N}_3 \upsilon^2 \tag{4}$$

Here $N_2 = (2r+1)/(2r+3)$, $\bar{N}_3 = (2r+3)/(2r+5)$, $\upsilon = \max|\dot{x}|$ and the r symbol represent the *linearization accuracy parameter*. As shown in [25], it can be selected from the interval (0, 2).

Taking into account (2) and (3), equations (1) take the form

$$m\ddot{x} + k_0 \dot{x} + c_0 x = T_0(\operatorname{sgn} U + B_T + k_T \dot{x}) - bx\cos \nu t - k_\eta \dot{x}_\eta - c_\tau x_\tau \tag{5}$$

$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T_0 (\operatorname{sgn} U + B_T + k_T \dot{x})$$

The solution of the equation with linearization can be constructed by the *method of change of variables with averaging* [25], which makes it possible to consider stationary and non-stationary processes. In [25], for a linearized general equation, a standard form of relations for these processes was obtained. In accordance with this standard form, we can immediately write out the results of solving the first equation of (5), and for the second equation we use the averaging procedure described in [29].

Equation solutions. Note that there are two fundamentally different cases, determined by the characteristic of the friction force $T(U)$ at $U > 0$, $u \ge ap$ and $U < 0$, $u < ap$, where $u = r_0 \Omega$ and Ω is the average value of the speed $\dot{\varphi}$ of the energy source. To derive the relations for $u < ap$, we use the technique described in [34].

By the method of change of variables with averaging noted above, we have

$$x = a\cos\psi, \ \dot{x} = -\upsilon \sin\psi, \ \psi = pt + \xi \tag{6}$$

where $\upsilon = ap$, $p = \nu/2$.

Taking into account $x_\tau = a\cos(\psi - p\tau)$, $\dot{x}_\eta = -\upsilon\sin(\psi - p\eta)$, we obtain from (5) the following equations for determining the non-stationary values of the amplitude *a*, phase ξ and velocity *u*:

a) $u \ge ap$:

$$\frac{da}{dt} = -\frac{1}{4pm}(2aA - ba\sin 2\xi)$$

$$\frac{d\xi}{dt} = \frac{1}{4pma}(2aE + ab\cos 2\xi) \tag{7,a}$$

$$\frac{du}{dt} = \frac{r_0}{I}\left[M(\frac{u}{r}) - r_0 T_0 (1+B_T)\right];$$

b) $u < ap$:

$$\frac{da}{dt} = -\frac{1}{4pm}\left[2aA - ba\sin 2\xi - \frac{8T_0}{\pi ap}\sqrt{a^2p^2 - u^2}\right]$$

$$\frac{d\xi}{dt} = \frac{1}{4pma}(2aE + ab\cos 2\xi) \tag{7,b}$$

$$\frac{du}{dt} = \frac{r_0}{I}\left[M(\frac{u}{r}) - r_0 T_0 (1+B_T) - \frac{r_0 T_0}{\pi}(3\pi - 2\psi_*)\right]$$

Here $A = p(k_0 + k_\eta \cos p\eta - T_0 k_F) - c_\tau \sin p\tau$, $E = m(\omega_0^2 - p^2) + c_\tau \cos p\tau$, $\omega_0^2 = c_0/m$, $\psi_* = 2\pi - \arcsin(u/ap)$.

Under the conditions $\dot{a} = 0$, $\dot{\xi} = 0$, $\dot{u} = 0$, equations of stationary motions are obtained from (7). In the case of $u < ap$, the amplitude of stationary oscillations is determined by the approximate expression $a \approx u/p$.

In the case of $u \geq ap$, the amplitude and phase of stationary oscillations are determined by the following expressions

$$A^2 + E^2 = 0.25b^2, \quad tg2\xi = -A/E \tag{8}$$

The stationary values of the velocity are found from the condition $\dot{u} = 0$, which gives the relation

$$M(u/r_0) - S(u) = 0 \tag{9}$$

Here the function $S(u)$ represents the load on the energy source and has the form

a) $u \geq ap$ $\rightarrow$ $S(u) = r_0 T_0 (1+B_T)$

b) $u < ap$ $\rightarrow$ $S(u) = r_0 T_0 \left[(1-B_T) + \pi^{-1}(3\pi - 2\psi_*)\right]$

The calculated stationary values of the amplitude are used to construct the $S(u)$ curve. The point (s) of intersection of the curves $M(u/r_0)$ and $S(u)$ determines the stationary value of the velocity u. In the case of $u < ap$, the expression of the load on the energy source $S(u)$ is simplified by taking into account the approximate equality $ap \approx u$ for the amplitude.

Stability of stationary oscillations. To test the stability of stationary motions, we compose equations in variations for (7), use the Routh-Hurwitz criteria, and obtain

$$D_1 > 0, \quad D_3 > 0, \quad D_1 D_2 - D_3 > 0 \tag{10}$$

where $D_1 = -(b_{11} + b_{22} + b_{33})$,

$$D_2 = b_{11}b_{33} + b_{11}b_{22} + b_{22}b_{33} - b_{23}b_{32} - b_{12}b_{21} - b_{13}b_{31}$$

$$D_3 = b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32}$$

For $u \geq ap$, we have:

$$b_{11} = \frac{r_0}{I}\left[Q + r_0 T_0 (\alpha_1 - 3\alpha_3 u^2 - 3\alpha_3 N_2 a^2 p^2) \right]; \quad b_{12} = -\frac{r_0^2}{I} 6\alpha_3 T_0 N_2 u a p^2; \quad b_{13} = 0$$

$$b_{21} = -\frac{1}{m} 3\alpha_3 T_0 a u; \quad b_{22} = -\frac{1}{m}\alpha_3 T_0 \bar{N}_3 a^2 p^2; \quad b_{23} = -\frac{Ea}{pm}$$

$$b_{31} = 0; \quad b_{32} = 0; \quad b_{33} = -\frac{A}{pm}$$

where $h = 3(u_0^2 - u^2)/\bar{N}_3$; $u_0^2 = \alpha_1/3\alpha_3$; $N_2 = (2r+1)/(2r+3)$; $\bar{N}_3 = (2r+3)/(2r+5)$ and $Q = \frac{d}{du} M(\frac{u}{r})$.

In the case of $u < ap$, the coefficients b_{11}, b_{12}, b_{21}, b_{22} take the form:

$$b_{11} = \frac{r_0}{I}\left[Q + r_0 T_0 (\alpha_1 + 3\alpha_3 u^2 + 3\alpha_3 N_2 a^2 p^2) - \frac{2 r_0 T_0}{\pi\sqrt{a^2 p^2 - u^2}} \right]$$

$$b_{12} = -\frac{2 r_0^2 T_0 u}{I}\left[3 N_2 \alpha_3 a p^2 + \frac{1}{\pi a \sqrt{a^2 p^2 - u^2}} \right]$$

$$b_{21} = -\frac{T_0 u a}{m}\left[3\alpha_3 - \frac{2}{\pi a^2 p^2 \sqrt{a^2 p^2 - u^2}} \right]$$

$$b_{22} = -\frac{T_0}{m}\left[\alpha_3 \bar{N}_3 a^2 p^2 + \frac{2u^2}{\pi a^2 p^2 \sqrt{a^2 p^2 - u^2}} \right]$$

Calculations. In order to obtain information about the effect of the delay on the oscillation modes, calculations were carried out with the parameters: $\omega_0 = 1c^{-1}$, $m = 1\,\text{kgf}\cdot\text{c}^2\cdot\text{cm}^{-1}$, $b = 0.07\,\text{kgf}\cdot\text{cm}^{-1}$, $c_0 = 0.05\,\text{kgf}\cdot\text{cm}^{-1}$, $k_0 = 0.02\,\text{kgf}\cdot\text{c}\cdot\text{cm}^{-1}$, $c_\tau = 0.05\,\text{kgf}\cdot\text{cm}^{-1}$, $k_\eta = 0.06\,\text{kgf}\cdot\text{c}\cdot\text{cm}^{-1}$, $T_0 = 0.5$ kgf, $\alpha_1 = 0.84\,\text{c}\cdot\text{cm}^{-1}$, $\alpha_3 = 0.18\,\text{c}^3\cdot\text{cm}^{-3}$, $r_0 = 1\,\text{cm}$, $I = 1\,\text{kgf}\cdot\text{c}\cdot\text{cm}^2$. For the delays $p\eta$ and $p\tau$, the values from the interval $(0,2\pi)$ were used, and in the linearization coefficients $N_2 = 3/5$, $\bar{N}_3 = 3/4$.

The amplitude-frequency curves $a(p)$ in Fig.2-4 were obtained at a speed u=1.2. The horizontal portions of the curves in Fig.4 reflect the dependence of $ap \approx u$. Curve *1* in all figures corresponds to the absence of delays ($k_\eta = 0$, $\eta = 0$, $\tau = 0$) and is shown for comparison. Oscillations with amplitudes are stable within the shaded and black-filled sectors for the steepness of the $Q = \frac{d}{du} M(u/r_0)$ characteristic of the power source. These sectors should be shown on the $S(u)$ load curve, but are shown on the amplitude curves for brevity. In the parts of the sectors filled with black, there is a rather weak stability, i.e. criteria (criterion) of stability (10) are fulfilled in the form $0.000\text{X} > 0$, where $\text{X} \le 9$.

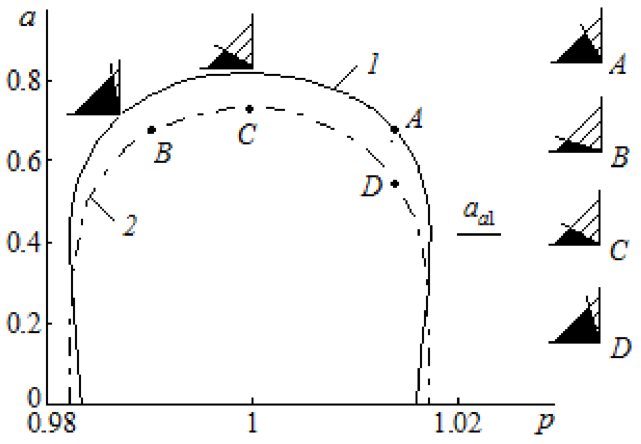

Fig.2. Amplitude-frequency curves at $\eta = 0$:
curve 1 – $\tau = 0$, curve 2 – $\tau = \pi/2$

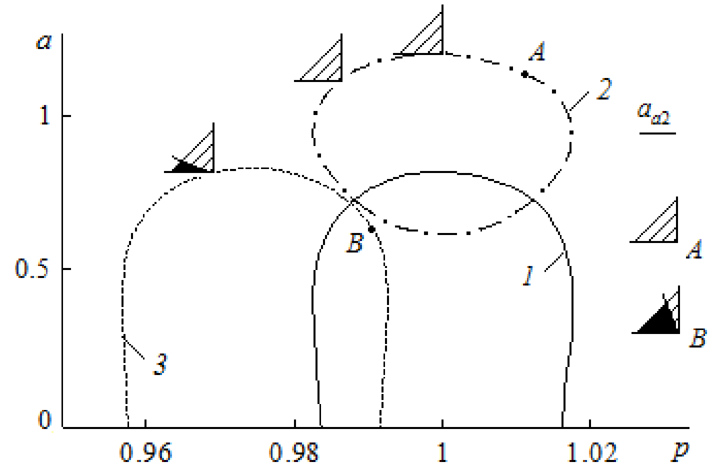

Fig.3. Amplitude-frequency curves at $\eta = \pi/2$:
curve 2 – $\tau = \pi/2$, curve 3 – $\tau = \pi$

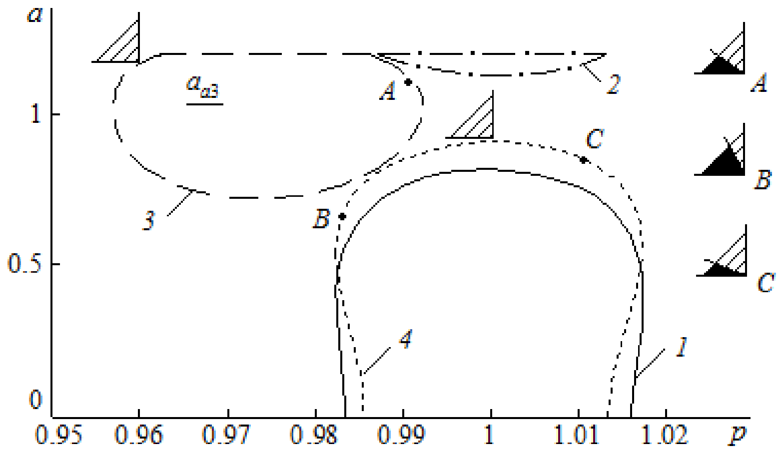

Fig.4. Amplitude-frequency curves at $\eta = \pi$:
curve 2 – $\tau = \pi/2$, curve 3 – $\tau = \pi$, curve 4 – $\tau = 3\pi/2$

Conclusion. Containing the linearization accuracy parameter, the method direct linearization allows you to easily obtain solutions to a nonlinear system of differential equations and derive relationships for calculating the values of the amplitude and phase of oscillations, as well as the speed of the energy source. The calculations performed show that the combined action of various combinations of elasticity retardation and damping can strongly influence resonant oscillations. Under such action, the resonant region can shift in frequency. Depending on various combinations of lag values, the amplitude of the oscillations increase/decrease, the stability of the oscillations increase/decrease.

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