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# PROCESSING OF TIME SIGNALS IN A DISCRETE TIME DOMAIN

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**Abstract:** This article is devoted to the processing of time signals in a discrete time domain. Time signals are the main object of analysis in many areas, such as signal processing, communication, control and much more. Today, for efficient signal processing, it is necessary to use methods adapted to the discrete time domain, using the Z-transform method to solve difference equations in the discrete time domain. The Z-transform method is a powerful and most effective tool for analyzing and solving difference equations, widely used in control systems, signal processing and other fields. The main steps of applying the Z-transformation method are also presented, starting from the formulation of the difference equation to obtaining a solution in the original time domain. Special attention is paid to the Z-transformation process, where the difference equation turns into an algebraic equation with respect to the Z operator. The basic properties of the Z-transformation and the operations with them necessary for the successful application of the method are described.

***Keywords:*** *temporary domain, discrete form, transformations, geometric progression, difference equation.*

**Introduction.** Processing of time signals is one of the key tasks in many areas, including signal processing, communication, control and many others. Effective signal processing is based on the use of methods adapted to the discrete time domain [1,2].

In the discrete time domain, the signals are presented in a discrete form consisting of a sequence of samples. This allows the use of various algorithms and processing methods specially designed to work with such discrete signals [3].

In the constant striving for the development and improvement of technologies in the modern scientific community, the study and application of various methods for solving difference equations is relevant. One of such methods that attracts considerable attention of researchers and engineers is the Z-transformation method. In this article we present an exhaustive analysis and new results obtained using this method [4].

The problem of solving difference equations is to find an analytical expression for the dependence between successive values of discrete variables in a discrete time domain. The Z-transform method allows you to move from a discrete time domain to a complex frequency domain using the Z operator. It provides an effective tool for analyzing and designing various systems based on difference equations [5-9].

In the course of our research, we conducted an in-depth analysis of the basic principles and theoretical aspects of the Z-transformation method. We have studied its properties, established convergence and stability conditions, and developed new methods and approaches for solving difference equations using this method [10-15].

The main results of our research include the development of algorithms for solving difference equations by the Z-transform method, analysis of their accuracy and stability, as well as optimization of the solution process using modern computing technologies.

One of the key aspects of processing time signals in a discrete time domain is sampling and quantization. Sampling is the process of splitting a continuous time signal into discrete samples. Quantization, in turn, refers to the process of converting analog signal values into a finite number of levels.

The main tool for processing time signals in a discrete time domain is the Fourier series transformation. The Fourier transform makes it possible to analyze the spectral characteristics of a signal, isolate its component frequencies and filter the signal in the frequency domain. To quickly calculate the Fourier transform, there is a fast Fourier transform algorithm that significantly reduces computational complexity.

Another method of processing time signals in a discrete time domain is the discrete cosine transform, which is widely used in data compression, image and sound compression, as well as in other areas where signal analysis and processing are required.

In addition, in the field of processing time signals in a discrete time domain, methods of spectral analysis, power spectral density estimation, time characteristics extraction and many other methods and algorithms are actively used. The use of these methods makes it possible to achieve more accurate analysis and processing of time signals, as well as to reveal hidden features and information contained in these types of signals.

One of the areas where processing of time signals in a discrete time domain is widely used is digital audio processing. With the help of appropriate methods, it is possible to process audio signals, including noise reduction, equalization, compression and other operations.

Another area of application of time signal processing in the discrete time domain is image processing. Using the appropriate algorithms, you can perform filtering, sharpening, segmentation and other operations on images. It finds application in the fields of computer vision, medical diagnostics, as well as in various tasks of image processing and analysis.

Along with these areas, the processing of time signals in a discrete time domain also finds application in radio communications, radar, signal management and many other areas where it is necessary to analyze, process and transmit signals using digital methods.

In addition, processing time signals in a discrete time domain has its advantages over analog signal processing. It allows the use of digital processing methods that provide higher stability, accuracy and flexibility when working with signals [16,17].

One of the key components of processing time signals in a discrete time domain is the conversion of time signals into sequences of numbers. This is done by sampling, where the signal is split into discrete samples, which can then be processed by digital algorithms [19].

After sampling, the signal can be represented in the form of a sequence of numbers that can be processed using various methods and algorithms. An important tool in this process is the Z-transform, which allows you to analyze and process signals in a discrete time domain.

The Z-transform method is a powerful tool for analyzing and processing time signals. It allows you to represent a time signal in the form of a Z-transformation, which is expressed in terms of coefficients and powers of the Z operator. Then, using algorithms and methods based on the Z-transformation, various operations can be performed with signals, such as filtering, deconvolution, spectrum estimation, and others [20,21].

The advantages of the Z-transform method include its versatility and applicability to various types of signals, the ability to accurately model and analyze signal systems, as well as ease of use and understanding. This makes the Z-transform method one of the main tools in the field of time signal processing [22,23].

**Problem statement.** Thus, our research is devoted to the deep analysis and development of the Z-transformation method for solving difference equations. In which the results obtained are an important contribution to the field of theory and practical application of this method.

Our work is aimed at improving the accuracy, efficiency and reliability of solving difference equations using this method.

By analyzing and optimizing Z-transformation algorithms to improve speed and computational efficiency. We found that this allowed us to reduce the calculation time and improve the performance of systems based on difference equations.

Secondly, by applying this Z-transformation method in combination with other methods and algorithms, such as the finite difference method, the finite element method or artificial intelligence methods. it allowed us to create more complex and powerful tools for solving complex problems and optimizing systems.

And finally, we are going to expand the scope of the Z-transformation method to new areas, such as discrete-time systems, nonlinear systems and systems with variable coefficients [24]. This will allow us to explore new aspects and possibilities of this method and unlock its potential in a wider range of applications.

The Z-transform method is one of the effective methods for solving difference equations in a discrete domain. To solve the difference equation by the Z-transformation method, the following steps follow [25]:

We set the difference equation in discrete form, for example:

$$\mathrm{a}_n \mathrm{y}[n] + \mathrm{a}\{n-1\}\ \mathrm{y}[n-1] + \ldots + \mathrm{a}_0\mathrm{y}[0] = \mathrm{b}_n\mathrm{x}[n] + \mathrm{b}_{\{n-1\}}\ \mathrm{x}[n-1] + \ldots + \mathrm{b}_0\mathrm{x}[0] \tag{1}$$

Here $\mathrm{y}[n]$ and $\mathrm{x}[n]$ are the values of the function $\mathrm{y}(\mathrm{t})$ and $\mathrm{x}(\mathrm{t})$ at discrete time points $n$, $a_{\mathrm{i}}$ and $b_{\mathrm{i}}$ are the coefficients of the difference equation.

We apply the Z-transform to both parts of the equation:

$$\mathrm{a}_n\mathrm{Y}(\mathrm{z})\ \mathrm{z}^n + \mathrm{a}_{\{n-1\}}\mathrm{Y}(\mathrm{z})\ \mathrm{z}^{\{n-1\}} + \ldots + \mathrm{a}_0\mathrm{Y}(\mathrm{z}) = \mathrm{b}_n\mathrm{X}(\mathrm{z})\ \mathrm{z}^n + \mathrm{b}_{\{n-1\}}\ \mathrm{X}(\mathrm{z})\mathrm{z}^{\{n-1\}} + \ldots + \mathrm{b}_0\mathrm{X}(\mathrm{z}) \tag{2}$$

Here $\mathrm{Y}(\mathrm{z})$ and $\mathrm{X}(\mathrm{z})$ are the Z-transformations of $\mathrm{y}[n]$ and $\mathrm{x}[n]$, respectively.

We express the Z-transformation of the desired function $\mathrm{Y}(\mathrm{z})$ through the Z-transformation of the input function X(z) and the initial conditions:

$$\mathrm{Y}(z) = (\mathrm{b}_n\ \mathrm{X}(\mathrm{z})\ \mathrm{z}^n + \mathrm{b}_{\{n-1\}}\mathrm{X}(\mathrm{z})\ \mathrm{z}^{\{n-1\}} + \ldots + \frac{\mathrm{b}_0\mathrm{X}(\mathrm{z})}{(\mathrm{a}_n\mathrm{z}^n + a_{\{n-1\}}z^{\{\mathrm{n}-1\}}} + \ldots + \mathrm{a}_0 \tag{3}$$

Using the table of Z-transformations and the properties of the algebra, we find the inverse Z-transformation for the resulting expression Y(z) in order to obtain a solution of y[$n$] in the time domain.

The inverse Z-transformation can be performed using methods such as the method of partial fractions, the method of decomposition into the simplest fractions or tables of Z-transformations.

The application of the Z-transformation method makes it possible to transfer the difference equation from the time domain to the Z-domain, where its solution can be found using algebraic operations and knowledge of the Z-transformation table. The result will be the solution of the equation in discrete moments of time.

Considering the simplest equation, which is the sum of the geometric progression using this method:

$$\mathrm{x}[n] = 2^n \tag{4}$$

To solve this equation by the Z-transformation method, we apply the following approach:

Apply the Z-transformation to both sides of the equation:

$$\mathrm{Z}\{\mathrm{x}[n]\} = \mathrm{Z}\{2^n\} \tag{5}$$

We use the well-known Z-transformation of the geometric progression:

$$Z\left\{a^n\right\} = \frac{1}{\left(1 - az^{(-1)}\right)},\ \textit{npu}\ |z| > |a| \tag{6}$$

Applying this transformation to the right side of the equation, we get:

$$X(z) = \frac{1}{\left(1 - 2z^{(-1)}\right)}; \tag{7}$$

Now we have an expression for $X(z)$. To get the inverse transformation, we find the decomposition into rational fractions:

$$X(z) = \frac{1}{\left(1 - 2z^{(-1)}\right)} = \frac{1}{\left(1 - \frac{2}{z}\right)} = \frac{1}{(z-2)} \tag{8}$$

We apply the inverse Z-transform to find the solution of the original equation:

$$x(n) = Z^{\{-1\}}\left\{X(z)\right\} = Z^{\{-1\}}\left\{\frac{1}{(z-2)}\right\} \tag{9}$$

Thus, the solution of this equation of the sum of the geometric progression will be this sequence. Graphically, the sequence x[*n*] will tend to infinity even faster than in the previous case. The growth will be exponentially accelerated and more pronounced. This is due to the fact that the exponent in the equation has become much larger, which leads to a sharper increase in each element of the sequence.

**Solving the problem.** We set the difference equation in discrete form, for example:

Find the solution of the difference equation by the z-transformation method

$$2y_k - 3y_{k-1} + 1{,}12y_{k-2} = 2 \times 1(k) \tag{10}$$

under zero initial conditions $\left(y_{-1} = 0 \text{ и } y_{-2} = 0\right)$

Solving this equation (10) to Z-images taking into account the given zero initial conditions. If $Z\left\{y_k\right\} = y(z)$, then according to the lag theorem we have:

$$Z\left\{y_{k-1}\right\} = z^{-1}y(z),\ Z\left\{y_{k-2}\right\} = z^{-2}y(z) \tag{11}$$

According to the transformation table, we also find $Z\left\{1[k]\right\} = \frac{z}{(z-1)}$. Substituting z-transformations instead of variables in (10) and grouping similar terms, we get

$$\left(2 - 3z^{-1} + 1{,}12z^{-2}\right)y(z) = 2\frac{z}{z-1}$$

Hence follows

$$y(z) = \frac{z^3}{(z-1)\left(z^2 - 1{,}5z + 0{,}56\right)} \tag{12}$$

Now, to solve the equation, it is enough to find the original of the resulting expression. To this end, we will take out *z*, and decompose the remaining fraction into the simplest fractions. The roots of the equation $z^2 - 1,5z + 0,56 = 0$ are equal to $z_1 = 0,7, z_2 = 0,8$. Therefore, the right part (12) can be represented as

$$z\left(\frac{z^2}{(z-1)(z-0,7)(z-0,8)}\right) = z\left(\frac{A}{z-1} + \frac{B}{z-0,7} + \frac{C}{z-0,8}\right) \tag{13}$$

The undefined coefficients in this equation will be found:

$$A = \left.\frac{z^2}{(z-0,7)(z-0,8)}\right|_{z=1} = \frac{1}{0,3\cdot 0,2} = 16,67,$$

$$B = \left.\frac{z^2}{(z-1)(z-0,8)}\right|_{z=0,7} = \frac{0,49}{-0,3\cdot(-0,1)} = 16,33,$$

$$C = \left.\frac{z^2}{(z-1)(z-0,7)}\right|_{z=0,8} = \frac{0,64}{-0,2\cdot 0,1} = -32.$$

Substituting the obtained values in (13) and then in (12), and we get

$$y(z) = 16,67\frac{z}{z-1} + 16,33\frac{z}{z-0,7} - 32\times\frac{z}{z-0,8} \tag{14}$$

In accordance with the transformation table, you can write correspondences:

$$\frac{z}{z-1} \to 1[k],\ \frac{z}{z-0,7} \to 0,7^k,\ \frac{z}{z-0,8} \to 0,8^k \tag{15}$$

Therefore, the solution of the given equation can have the form

$$y_k = 16,67\cdot 1[k] + 16,33\cdot(0,7)^k - 32\times(0,8)^k \tag{16}$$

The equation is the sum of three components: the first term 16.67×1[*k*] is constant and independent of *k*, the second term $16.33\times(0.7)^k$ is a geometric progression with decaying values, and the third term $-32\times(0.8)^k$ is also a geometric progression, but with increasing values.

The value of $y_k$ depends on the value of *k*. As *k* increases, the second term decreases, and the third term increases, which leads to a change in the overall value of $y_k$. The first term remains constant.

The dependence on the value of *k* can be used to model various processes where it is necessary to take into account decaying or growing values over time.

Graphically, the values of $y_k$ can be represented as a curve that combines a constant component and two geometric progressions with different rates of change. The shape of the curve will depend on the values of the coefficients and initial conditions, where a variety of dependencies is demonstrated and can be useful for modeling various processes where changes over time are important.

The results of the calculations can be visualized using a graph, where the value of the index $k$ will be plotted on the x axis, and the corresponding value $y_k$ will be plotted on the y axis. This will allow you to visualize the change in the sequence $y_k$ depending on the value of the index *k*. The graph of the resulting solution is shown in Fig.1.

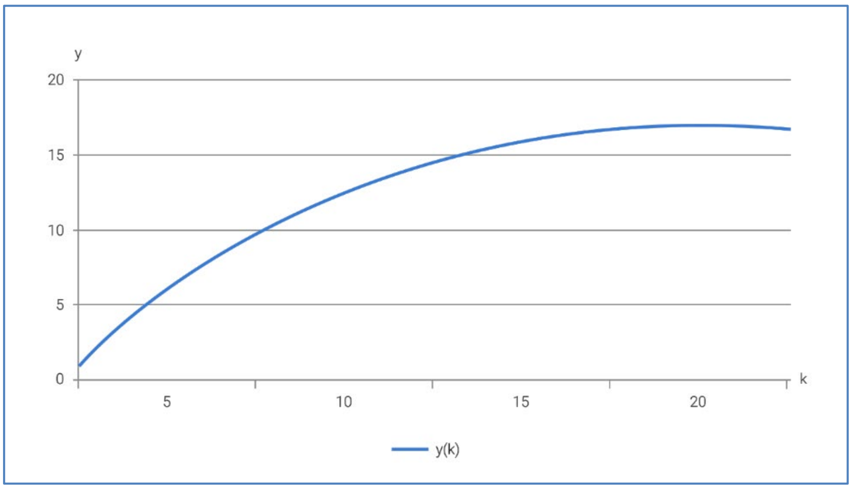


*Figure 1. Graph of the solution of the difference equation*

The same example is only for a system of differential equations

$$\dot{x} = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} g \tag{17}$$

$$y = \begin{bmatrix} 2 & 0 \end{bmatrix} x + 0,2g$$

for $x_0 = [1\ 1]^T$ and $g(t) = 1,5t, t \geq 0$, and plot the dependence of $y(t)$ and $g(t)$ in Figure 2

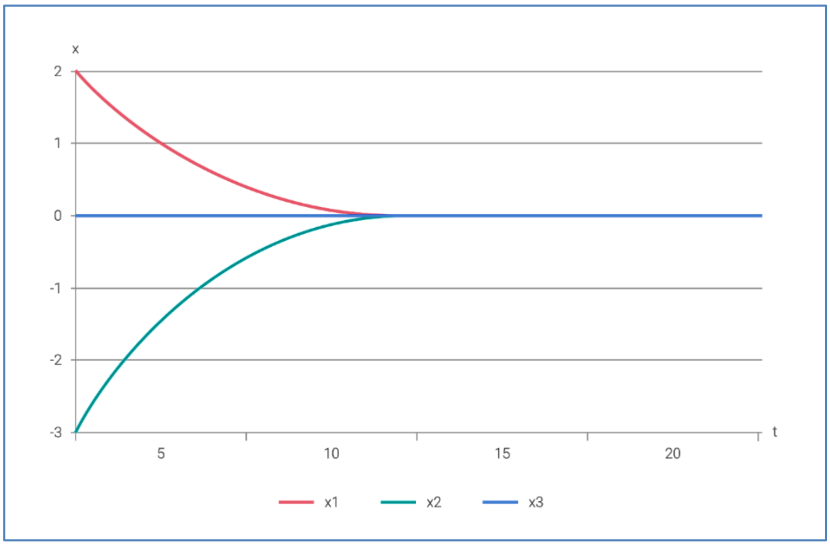


*Figure 2. Graph of variable states*

The given system of differential equations (17) is a special case of the system

$$\dot{x} = Ax + Bg, \tag{18}$$

$$y = Cx + Dg. \tag{19}$$

The solution of the system (18), (19) is determined by the Cauchy formula:

$$y = Ce^{At}x_0 + \int_0^t Ce^{A(t-\tau)}Bg(\tau)\,d\tau + Dg(t) \tag{20}$$

Thus, according to the formula (7), it is necessary to first find the matrix $e^{At}$. In the case of system (17), the matrix *A* coincides with the matrix for which the matrix $e^{At}$ (18) is found in example (12). Using this expression, we find that the free component of the solution (20) of system (17) will be equal to

$$y_{cB}(t) = Ce^{At}x_0 = \begin{bmatrix}2 & 0\end{bmatrix}\begin{bmatrix}2e^{-4t}+e^{-t} & 2e^{-t}-2e^{-4t} \\ e^{-t}-e^{-4t} & e^{-4t}+2e^{-t}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{3} = 2e^{-t} \tag{21}$$

To calculate the integral in (20), we first find the product

$$Ce^{A(t-\tau)}\mathrm{B} = \begin{bmatrix}4\mathrm{e}^{-4t}+2e^{-t} & 4e^{-t}-4e^{-4t}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\frac{1}{3} = \frac{4}{3}\left(e^{-t}-e^{-4t}\right) \tag{22}$$

Next, replacing $t$ by $t-\tau$ here, we write the integral of (20):

$$I = \int_0^t \frac{4}{3}\left(e^{-(t-\tau)} - e^{-4(t-\tau)}\right)1{,}5\tau d\tau = 2e^{-t}\int_0^t e^{\tau}\tau d\tau - 2e^{-4t}\int_0^t e^{4\tau}\tau d\tau. \tag{23}$$

Integrating in parts or applying the calculation for $\alpha = 1$ and $\alpha = 4$, we will have

$$I(t) = 2e^{-t}\left[e^{t}(t-1)+1\right] - 2e^{-4t}\left[e^{4t}\left(\frac{t}{4}-\frac{1}{16}\right)+\frac{1}{16}\right] = \frac{3}{2}t - \frac{15}{8} + 2e^{-t} - \frac{1}{8}e^{-4t} \tag{24}$$

Now, summing up, according to (7), all the components of the solution, we get:

$$y(t) = 2e^{-t} + \frac{3}{2}t - \frac{15}{8} + 2e^{-t} - \frac{1}{8}e^{-4t} + 0{,}2\cdot 1{,}5t = 1{,}8t - \frac{15}{8} + 4e^{-t} - \frac{1}{8}e^{-4t} \tag{25}$$

The graphs of the functions g(*t*) and y(*t*), taking into account the fact that the "slowest" exponent "fades" in this case in 3c, are shown in Fig. 3

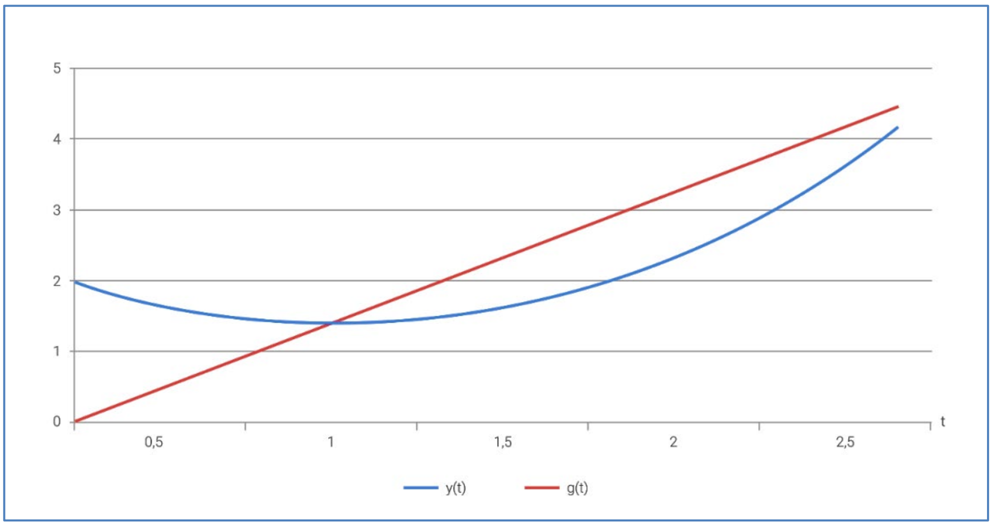


*Figure 3. Exciting function of the differential equation system*

The resulting solution has a component of $1{,}8t$ proportional to the input effect $g(t) = 1{,}5t$, and two exponents $\exp(-t)$ and $\exp(-4t)$, whose exponents are equal to the roots of the characteristic polynomial of the matrix $A$ of a given system of equations (17). All these exponents are eigenmodes of some dynamic the system that is described by these equations.

**Conclusion.** As a result of the conducted research and application of the Z-transformation in the field of time signal processing, the following complex conclusions can be drawn that when using the Z-transformation, signals can be represented in the form of difference equations, which allows for analysis and processing of signals in a discrete time domain.

The advantages of the Z-transform include its ability to represent complex time signals in a compact and convenient form, as well as to enable various signal processing operations, such as filtering, decomposition and reconstruction of signals, the Z-transform provides an effective tool for analyzing and processing time signals in a discrete time domain.

With the use of the Z-transform, a high degree of flexibility and accuracy can be achieved in the processing of time signals, thanks to the possibility of setting parameters and applying various processing methods. The use of the Z-transform makes it possible to solve various tasks of processing time signals, such as noise filtering, data compression, signal characterization, and others.

The results of the research and application of the Z-transform in this work allow us to assert that this method is an effective tool for analyzing and processing time signals, and its application can lead to significant results in various fields of application.

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