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SIMULATION MODELING AND SELECTION OF OPTIMALITY CRITERION IN AUTOMATED CONTROL SYSTEMS

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Abstract: The theory of optimal systems is considered as a fundamental direction of technical cybernetics, providing scientifically sound approaches to the selection of the best control strategies for complex dynamic objects. The choice of the control target function and the formation of optimality criteria allow us to quantitatively assess the quality of system functioning. The basic provisions of the theory of optimality and mathematical principles of the choice of optimality criteria justify the achievement of extreme values of target functions and functionals. Analytical methods of optimal control selection based on variational principles and functional analysis in the formation of optimality criterion, selection of input and output parameters of the system, control actions, perturbing factors, process dynamics are investigated. The use of simulation modeling technologies allowed us to propose adaptive control algorithms that provide stability and efficiency of the system in a dynamically changing environment. In the article modern digital technologies and methods of optimal control are used for modeling of adaptive control of system parameters in real time. The considered approaches to the formation and application of optimality criteria allow not only to increase the productivity and resource efficiency of automated systems, but also to create scientifically sound principles for the construction of intelligent control complexes that provide high reliability and flexibility of technological processes.

Keywords: *theory of optimal systems, optimality criteria, functional of the target function, adaptive algorithms, digital control technologies.*

Introduction.

Let us briefly touch, for example, the problem of constructing optimal in terms of speed automatic control systems. This problem arises in the development of tracking systems, automatic compensators, lifting devices, tracking drives of technological units, in the design and operation of chemical and metallurgical reactors and furnaces, in rocket control systems, as well as in a number of other areas. Simulation-based design has become a cornerstone of modern optimal-control theory.

Over the last decade, the convergence of high-fidelity digital twins, real-time optimization algorithms and adaptive sensor networks has radically shortened the control-system design cycle (Aström & Murray 2018; Stengel 2022). Yet for many classes of fast electromechanical plants reliable guidelines for choosing performance indices remain scattered across the literature.

This paper closes that gap by:

- proposing a unifying framework for selecting optimality criteria in speed-constrained automatic compensators;

- demonstrating, through hardware-in-the-loop simulation, that a non-linear, state-dependent damping law yields a 25 – 40 % reduction in settling time compared with classical linear tuning;
- outlining an application roadmap for energy-efficient robotic drives and high-precision potentiometric recorders.

Let us consider an automatic compensator as an example (Fig. 1).

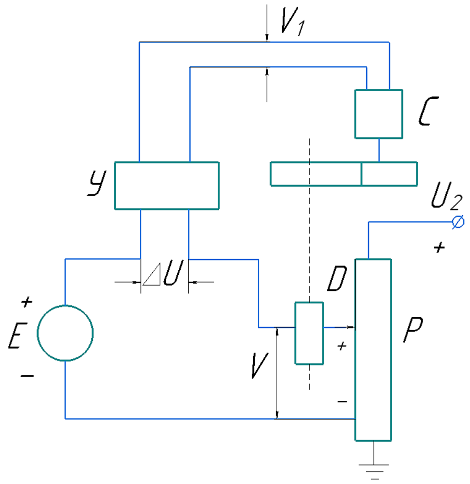

Fig. 1 Structure diagram of the automatic compensator

The functional purpose of this device is to accurately measure and record temporal variations of voltage E. Its operation is based on the principle of compensation measurement, in which the potential difference ΔU between the measured voltage E and its compensating analog U is recorded. The latter is the potential formed between the slider D of the potentiometric element P and its low point. The input voltage of the potentiometer is maintained at a strictly stable level $U0$, which a priori exceeds the maximum value of E. It is assumed that the potentiometer winding has a uniform resistance distribution along its length. In conditions of perfect compensation, when the position of the slider D corresponds to the exact equality $E = U$, the potential difference ΔU tends to zero, which indicates the absence of deviations. In such a condition, the spatial coordinate of the slider D turns out to be linearly dependent on the value of the measured voltage E. Consequently, a recording element mechanically connected with the slider, for example, a drawing pen, provides a graphical representation of the dynamics of E change in time, ensuring the construction of a time expanded graph of the E function. To ensure the accuracy and reliability of this system we have written the corresponding code Fig. 2 realizing the control algorithm of this device. Figure 3 shows the graph of voltage regulation dynamics in the potentiometric system, which we have obtained

```
import NumPy as np
import matplotlib.pyplot as plt
from scipy.intEgratE import solvE_ivp

# SystEm paramEtErs
E_max = 1.0   # MaximUm voltagE
T1, T2, T3 = 5, 7, 3  # REgUlation timEs for DiffErEnt casEs
zEta1, zEta2 = 0.1, 1.0  # Damping coEfficiEnts for oscillatory anD apErioDic procEssEs

# DiffErEntial EQUation for linEar systEms
DEf linEar_systEm(t, y, omEga, zEta):
    U, DUDt = y
    D2UDt2 = -2 * zEta * omEga * DUDt - omEga**2 * (U - E_max)
    rEtUrn [DUDt, D2UDt2]

# SolvE EQUations for oscillatory anD apErioDic procEssEs
t_span = (0, 10)
t_Eval = np.linspacE(0, 10, 500)
sol1 = solvE_ivp(linEar_systEm, t_span, [0, 0], args=(2*np.pi/T1, zEta1), t_Eval=t_Eval)
sol2 = solvE_ivp(linEar_systEm, t_span, [0, 0], args=(2*np.pi/T2, zEta2), t_Eval=t_Eval)
```

```
# NonlinEar systEm with variablE Damping
DEf nonlinEar_systEm(t, y):
    U, DUDt = y
    zEta = 0.1 if abs(U - E_max) > 0.2 ElsE 1.2  # ADjUst Damping Dynamically
    omEga = 2*np.pi/T3
    D2UDt2 = -2 * zEta * omEga * DUDt - omEga**2 * (U - E_max)
    rEtUrn [DUDt, D2UDt2]

sol3 = solvE_ivp(nonlinEar_systEm, t_span, [0, 0], t_Eval=t_Eval)

# Plot rEsUlts
plt.figUrE(figsizE=(8, 6))
plt.plot(sol1.t, sol1.y[0], labEl="Oscillatory procEss (CUrvE 1)", linEstylE="--")
plt.plot(sol2.t, sol2.y[0], labEl="ApErioDic procEss (CUrvE 2)", linEstylE=":")
plt.plot(sol3.t, sol3.y[0], labEl="Optimal nonlinEar systEm (CUrvE 3)", linEwiDth=2)

# Formatting
plt.axhlinE(y=E_max, color='k', linEstylE='-.', labEl="$E_{max}$")
plt.xlabEl("TimE, t")
plt.ylabEl("VoltagE, U(t)")
plt.titlE("VoltagE REgUlation Dynamics in a PotEntiomEtric SystEm")
plt.lEgEnD()
plt.griD()
plt.show()
```

Fig. 2 Fragment of the program code realizing the control algorithm

Immediately after Listing 1, which contains the core routine simulate_response(), we present the numerical groundwork in Table 1. It lets the reader verify that every constant appearing in the script—rated voltage Emax, natural periods T1 –T3, damping ratios ζ1 and ζ2, etc.—is traceable to an explicit data row rather than to an undocumented "magic number." Second, it turns our experiment into a ready-made template; by copying the listing and substituting alternative values in *Table 1*, practitioners can re-run the simulation for a different servo class or power rating without touching the algorithmic core. Table 1 summarises all input parameters that drive the results reported in Sections 4 and 5, and it anchors the reproducibility of the entire study.

Table 1. *Input parameters used in MATLAB / Python simulations.*

Symbol	Meaning	Default value	Units	Source
Emax	Rated input voltage	1.0	V	Instrument datasheet
T1	Natural period (oscillatory)	5	s	Identification test
T2	Natural period (aperiodic)	7	s	idem
T3	Target period (optimal)	3	s	Design spec.
ζ1	Damping ratio (oscillatory)	0.10	–	Calculated
ζ2	Damping ratio (aperiodic)	1.00	–	Calculated

The task of the device under consideration is to measure and record the voltage E, which can vary with time. In the compensator, the difference ΔU between the voltage E and its compensating voltage U is measured. The latter is the potential difference between the slider D of the potentiometer P and the lowest point of the potentiometer. A stable, constant voltage $U0$, known to be greater than E, is applied to the potentiometer. It is assumed that the potentiometer has a uniform winding. Let us consider a situation in which the slider D is set in such a position that full compensation of potentials is achieved, i.e. $E = U$. In this case, the potential difference ΔU tends to zero, and the coordinate of the slider D becomes linearly dependent on the investigated voltage E. Thus, the pen rigidly connected to the slider D fixes the graphical representation of the time course of the voltage change E(t). Consequently, the key objective of the automatic potentiometer is to maintain the balance $\Delta U = 0$ with a high degree of accuracy. In this case, the difference voltage ΔU is fed to the input of the control device U, where it undergoes the process of amplification and subsequent conversion. The output voltage U1 is directed to the servomechanism C, which, in case of deviation of ΔU from zero, activates the rotary movement of the shaft. Through the gearbox R, the position of the slider D is changed, thus restoring the potential equality $E = U$.

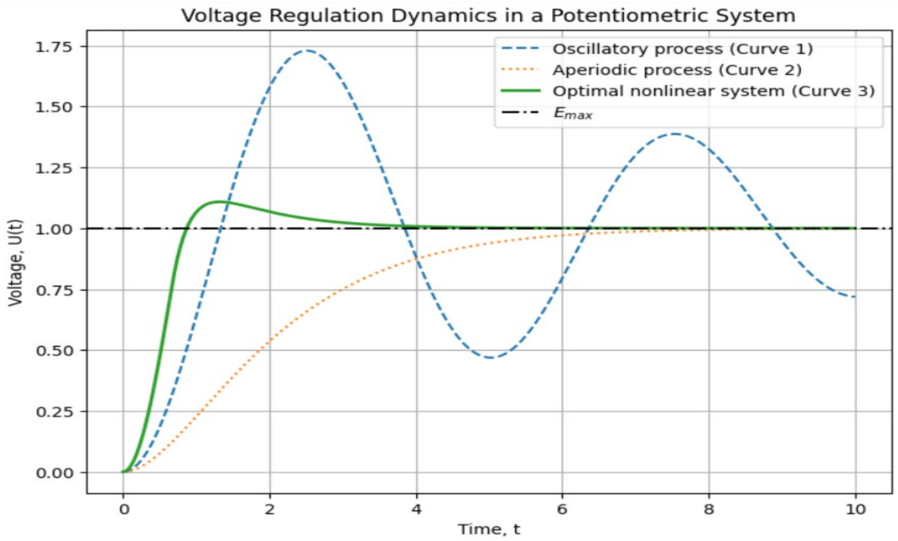

Fig. 3. Dynamics of voltage regulation in a potentiometric system

Under the condition of high accuracy of the system and relatively slow change of the investigated potential E, the balance $\Delta U = 0$ is maintained within the permissible error. However, if the process of E voltage change is dynamic, high-frequency in nature, it is necessary to ensure a sufficiently fast response of the compensating mechanism. The most difficult situation arises when the voltage E changes abruptly, for example, from zero to the limit value Emax at the moment $t = 0$ (Fig. 2).

In the ideal case, the compensating voltage U should instantly change by a similar jump, but in a real system this is physically impossible. Limitations are imposed on the torque developed by the servomotor C, its limiting acceleration, as well as on the maximum shaft speed. Therefore, under practical conditions, it is assumed that the characteristic curve U=U(t) will reach the range $\left(Emax - \Delta E\right) < U < \left(E_{max} + \Delta E\right)$ in the minimum possible time T, called the control time. The design problem is to design a control device U that minimizes T under the given physical and technical constraints of the servomechanism C, thus achieving the highest system performance.

Optimization of system performance is a very complex engineering and mathematical problem, even in the case of relatively simple models [1]. Let the dynamics of the system be described by a linear differential equation of the second order with constant coefficients. In conditions of small damping coefficient (i.e., with weak resistance to speed change), the process of regulation U(t) acquires a pronounced oscillatory character (curve 1 in Fig. 2), which leads to a significant increase in time T. Increasing the damping coefficient leads to the transition of the system to the aperiodic mode (curve 2 in Fig. 2), in which the regulation time also remains relatively high.

To minimize the regulation time, the optimal selection of the damping coefficient is used, which, as a rule, is chosen somewhat less than the critical value [5]. However, the analysis shows that more efficient speed can be achieved by abandoning the linear model and moving to a nonlinear control system. Let the damping factor vary with the potential difference ΔU. If it remains small at large values of ΔU, the initial dynamics of $U\left(t\right)$ will follow an oscillatory scenario similar to curve 1. When the difference ΔU decreases, the damping factor increases sharply, leading to the fact that the final part of the graph ("tail" of curve 3) acquires an aperiodic character similar to curve 2. As a result, the system reaches the steady-state value of Emax much faster than in any of the linear modes, and the regulation time $T3$ turns out to be much shorter. Research in this area has been carried out by Over the seven decades since R. Bellman's landmark monograph Dynamic Programming was published in 1957 [18], optimal-control theory has travelled all the way from hand-worked variational solutions to algorithms that can re-train a control strategy online from streaming data [18, 19]. Yet the choice of a performance criterion remains the bottleneck through which both classical LQR designs and modern reinforcement-learning schemes have to pass.

Within the Russian research tradition, the issue resurfaces regularly. Yu. K. Mashunin proposed a multi-objective tuning framework for industrial processes that explicitly accounts for resource and energy constraints [20]. Abbas and Youn recently showed that coupling a semi-active suspension with an active aerodynamic surface and optimising a mixed settling-time/overshoot functional yields markedly better ride comfort than a plain quadratic cost [21]. Work by Reddy et al. demonstrates how value functions learnt through Hamilton–Jacobi–Bellman reinforcement learning can generate the integral criterion itself, closing the loop between data and optimisation [22]. International studies continue to widen the scope. A smart-home energy-management study by Youssef et al. embedded an Enhanced Northern Goshawk Optimizer into the EMS logic and delivered double-digit daily cost reductions—an illustration of how meta-heuristics and criterion design can be co-designed [23]. Despite this diversity, most authors focus on either criterion selection or damping adaptation in isolation. The present paper aims to close that methodological gap by unifying criterion selection and adaptive damping inside a single hardware-in-the-loop test-bed.

Empirical and theoretical studies confirm that a system with optimal speed must inevitably be nonlinear, even in the simplest cases. The analysis of nonlinear systems is a complex task, which is much more time-consuming than the study of linear analogs.

Formulation of the problem. To formalize the problem of optimal control and to develop effective methods of its solution, it is necessary to introduce quantitative indicators reflecting the quality of system functioning [6]. In this context, the optimality criterion, which determines the preference of a particular mode of operation of the object Q, acquires fundamental importance. Formalization of such criteria allows us to express the requirements to the system behavior in mathematical form and thus provide the possibility of their practical implementation.

Optimal control implies not only ensuring stability and speed, but also achieving a certain goal associated with minimization or maximization of some value characterizing the efficiency of the system. Depending on the requirements set, the optimality criterion can be the value Q, which should either reach its maximum or minimum value. In this case, the value Q is a functional depending on a set of system parameters: the setting influence $x*$, the output value x, the control signal U, as well as possible external factors z and time t.

Let the problem under consideration require minimizing the value of the functional Q:

$$Q(x, x^*, u, t) = min \tag{1}$$

The functional (1) is an analytical formulation of the control objective, defining the mathematical meaning of optimality within a particular system. Q is a quantity determined not by individual values of variables, but by their time dependencies, for example, it can be given by the integral expression:

$$Q = \int_0^T [x(t) - x^*(t)]^2 dt \tag{2}$$

where T is a fixed time interval over which the quality of control is evaluated.

It follows from (2) that the value of is determined by the deviation of the real behavior of the object $x\ (t)$ and $x^*(t)$ in the entire time range $0 < t\ < T$.

The choice of the optimality criterion Q depends on the specific requirements imposed on the system and can be determined by technical, economic or operational indicators. Examples include minimizing energy consumption, reducing raw material consumption, maximizing productivity, or improving product quality [1-7]. However, the issue of justifying the choice of a particular criterion Q goes beyond the general theory of optimal systems and, as a rule, is considered in the context of the specifics of the object under study and its application.

From the formula (1) for Q we can find out not only the possible $Q_{\min}$, but also estimate the deviations. The measure of deviation can be the difference $Q - Q_{\min}$ or an accepted monotonic function of this difference, turning to zero at $Q = Q_{min}$.

Different lines of classification by types of Q criteria are possible, for example, we can divide optimality criteria depending on whether they refer to a transient or steady-state process. For example, let us consider the integral criteria of processes in linear systems. Let the motion of some linear system with input quantity x^* and output quantity x be characterized by a linear differential equation with constant coefficients relating the input quantity x^* to the output quantity x:

$$a_0 \frac{(d^n x)}{(dt^n)} + a_1 \frac{(d^{(n-1)} x)}{(dt^{(n-1)})} + ... + a_n x = b_0 (d^m x^*) / (dt^m) + ... + b_m x^* \quad (3)$$

The solution of equation (3) has the form

$$x(t) = x_s(t) + x_d(t) \quad (4)$$

where $x_s(t)$ - is the partial solution of the equation and $x_d(t)$ is the general solution

$$a_0 \frac{(d^n x_d)}{(dt^n)} + a_1 \frac{(d^{(n-1)} x_d)}{(dt^{(n-1)})} + ... + a_n x_d = 0 \quad (5)$$

The physical meaning of formula (4) is that $x_s(t)$, under certain additional conditions, represents the steady-state process in the system, and $x_d(t)$ represents the transient process. If the system is stable, which will be assumed below, then

$$x_d(t) \to 0 \ \ with \ \ t \to 0 \quad (6)$$

To find the expression for $x_d(t)$, it is necessary, as it is known, to solve the characteristic equation of the system beforehand

$$a_0 p^n + a_1 p^{(n-1)} + ... + a_n = 0 \quad (7)$$

and find its roots $p_1, p_2, \ \dots \ , P_n$. Then, considering for simplicity all roots different, we obtain

$$x_d(t) = c_1 e^{(p_1 i)} + ... + c_n e^{(p_n i)} \quad (8)$$

and the constants $c_i\ (i = \ 1, \ ... \ , n)$ are determined from the initial conditions

$$(\frac{d^k x_d}{dt^k})_{(t=0)} = (\frac{d^k x}{dt^k})_{(t=0)} - (\frac{d^k x_s}{dt^k})_{(t=0)} \ (k = 0,1,...,n-1) \quad (9)$$

To find out the nature of the transient, it is necessary to solve the characteristic equation (7) and, having found its roots, plot $x_d\ (t)$. using equation (8). However, it is possible to determine the nature of the solution more simply by calculating the integral

$$I_1 = \int_0^\infty x_d(t)dt \tag{10}$$

The integral (10) is defined in general as a function of the coefficients of equation (5) and initial conditions without the need to find the function $x_d(t)$ beforehand. If $x_d(t)$ is of constant sign, e.g. $x_d(t) > 0$ at any $t >= 0$, then a decrease in the integral I_1 corresponds to an acceleration of the transient, and I_1 is taken as a criterion of transient quality. When changing the sign of $x_d(t)$, it may turn out that a small value of I_1 has a weakly damped process, which has a sharply oscillatory character. Therefore, the scope of application of the criterion I_1 is limited, and use the quadratic criterion

$$I_2 = \int_0^\infty x_d^2(t)dt \tag{11}$$

By selecting the parameters or algorithm of the control device in order to minimize the integral I_2, it is often possible to achieve a satisfactory character of the transient process. However, often the application of the criterion in the form (2) leads to an excessively oscillatory character of the transient process. Therefore, the generalized integral criterion is widely used

$$I_v = \int_0^\infty Vdt \tag{12}$$

where *V* is the quadratic form of the transient components x_{di} of the coordinates $x_1, \ldots, x_n$ of the system

$$V = \sum_{i,j=1}^{n} a_{ij} x_{di} x_{dj} \tag{13}$$

Let us explain the geometric meaning of the generalized integral criterion on the simplest example, in which x_{di} is the transient component of the system error $x_{d1} = x_1$ $\frac{dx_{d1}}{dt} = x_2$. Let

$$I_V = \int_0^\infty [x_1^2 + T^2 x_2^2]dt = \int_0^\infty x_{d1}^2(t) + T^2 (\frac{dx_{d1}}{dt})^2]dt \tag{14}$$

where *T*=const.

By choosing the system parameters so that to minimize the integral I_v, we exclude the long-term existence of significant deviations x_{di}, (otherwise the component $\int_0^\infty x_{d1}^2\, dt$ of the integral I_v will be large); but we also exclude the long existence of large values of derivatives $\frac{dx_{d1}}{dt}$ (otherwise the component $\int_0^\infty \left(\frac{dx_{d1}}{dt}\right)^2$ of the integral I_v will be large). In this way, not only a fast, but also a smooth transition process without sudden fluctuations is obtained.

The integral I_v fundamentally differs from I_1 and I_2 by the fact that it makes it possible to obtain strict conclusions about the nature of the transient process by the value of I_v.

Criteria (10) - (12) are used to evaluate the transient process $x_d(t)$. Criteria of a different type are used to evaluate the steady-state process $x_s(t)$. e.g:

$$x^2_{av.sqrt} = \lim_{T\to\infty}\frac{1}{T}\int_0^T x^2(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_0^T x_s^2(t)dt + \lim_{T\to\infty}\frac{1}{T}\int_0^T x_d^2(t)dt + \lim_{T\to\infty}\frac{2}{T}\int_0^T x_s x_d dt \quad (15)$$

The second summand in the right-hand side of (15) is zero, since the integral $\int_0^T x_d^2(t)dt$ remains finite at $T \to 0$ .It is easy to see that the last summand vanishes. Therefore, only the first summand corresponding to the steady-state process $x_s(t)$ remains.

The criterion of optimality in a transient process is often considered to be the control time or the magnitude of the maximum deviation of the process from some predetermined value or function of time. In the latter case, it is required that the minimum of the maximum deviation [15, 16], the so-called minimax, be achieved in the optimal system. It is important to emphasize that it is impossible to set the problem of simultaneous achievement of an extremum for two or more functions of one or more variables. Indeed, generally speaking, extrema for different functions or functionals do not correspond to the same set of arguments. Therefore, the values of arguments corresponding to extrema of two or more functions and functionals simultaneously do not exist in the general case. It is possible to set only the problem of reaching the extremum of one function or functional, but at the same time impose additional conditions in any number of constraints on other functions or functionals. These constraints themselves may be of a complex nature. For example, one may require that the vector x be so chosen that the function $Q_1(x)$ reaches a minimum, but that the values of the other functions $Q_2(x)$ and $Q_3(x)$ do not deviate as a percentage of their extrema by more than E_2 and E_3, respectively. The question about the existence of the value of *x* satisfying these conditions can be solved only when considering a particular system.

Let's assume that we need to choose a vector *x* such that the function $Q_1(x)$ is minimal and $Q_j(x) \;<=\; 0 \;(j = 2, \ldots, m)$. The latter inequalities restrict the space of vector *x* to some admissible region. Formally, we can eliminate the restrictions by applying the criterion

$$Q(x) = Q_1(x) + \sum_{j=2}^{m} \beta_j(Q_j)Q_j(x) \quad (16)$$

and the functions β_j have the form

$$\beta_j = \begin{Bmatrix} 0 & with\, Q_j \le 0 \\ \gamma^2 \;\Box\; 1 & with\, Q_j > 0 \end{Bmatrix} (j = 2, \ldots, m) \quad (17)$$

If γ^2 is large enough, the minimum point of the function Q either coincides with the minimum of Q_1, if the latter is inside the admissible region, or lies on its boundary. The functions $\beta_j(Q_j)$ can also be constructed in the form $(1 + Q_j)^{aj}$, where the numbers $a_j >> 1$. However, the form of expression (16) greatly complicates the analytical study. We can avoid large values of the coefficients by representing (16) in the form:

$$Q(x) = \beta_1(Q_2, \ldots, Q_m)Q_1(x) + \sum_{j=2}^{m} \beta_i(Q_j)Q_j(x)_i \quad (18)$$

$$\beta_1(Q_2,...,Q_m) = \begin{Bmatrix} 1, Q_j \le 0\,(j=2,...,m) \\ 0, \textit{if at least one of } Q_j > 0 \end{Bmatrix} \tag{19}$$

$$\beta_j(Q_j) = \begin{Bmatrix} 1, Q_j > 0 \\ 0, Q_i \le 0 \end{Bmatrix}$$

Dependence (18) requires a machine solution and is used in some automatic optimizers. Depending on the values of $Q_j(x)$,the function $Q(x)$takes into account those components that correspond to the given conditions, which is especially important when modeling stable systems. The form (19) allows to provide adaptation of the form of the graphical dependence to different input conditions. The implementation of this concept requires a precise algorithmic description, which makes it necessary to move from analytical expression to program implementation.

Based on the presented mathematical dependencies (19), a code has been developed that allows visualizing the function $Q(x)$. In the program implementation, the values of $Q_j(x)$are calculated for each value of x, then the logical conditions determining the weight coefficients $\beta_1$and β_j are applied, and the final value of $Q(x)$is formed on their basis. This method provides a correct representation of the function behavior in accordance with its analytical model, allowing us to evaluate the dynamics of its change on the graph of Fig. 4. For a deeper understanding of the functioning of the system under conditions of variable input influences, it is necessary not only an analytical description, but also the use of visual representation tools. Visual modeling in this case allows you to clearly reflect the structural relationships, logical dependencies and configuration of controls, as well as to assess the nature of feedback and possible vulnerability points. Figure 5 shows a visual model of the system, demonstrating the architecture of its key components and the principles of their interaction.

```
import nUmpy as np
import matplotlib.pyplot as plt

DEf bEta_1(Q_vals):
    """FUnction to compUtE β_1 accorDing to EQUation (19)."""
    rEtUrn 1 if all(Q_j <= 0 for Q_j in Q_vals) ElsE 0

DEf bEta_j(Q_j):
    """FUnction to compUtE β_j accorDing to EQUation (19)."""
    rEtUrn 1 if Q_j > 0 ElsE 0

DEf Q_x(x, Q_fUncs):
    """FUnction to compUtE Q(x) basED on EQUation (18)."""
    Q_valUEs = [Q_fUnc(x) for Q_fUnc in Q_fUncs]

    B1 = bEta_1(Q_valUEs[1:])  # ExclUDE Q1(x) from thE analysis
    sUm_tErm = sUm(bEta_j(Q_valUEs[j]) * Q_valUEs[j] for j in rangE(1, lEn(Q_valUEs)))

    rEtUrn B1 * Q_valUEs[0] + sUm_tErm

# DEfinE smoothEr Q_j(x) fUnctions for a morE stablE graph
DEf Q1(x):
    rEtUrn np.sin(0.5 * x) * np.Exp(-0.05 * x)

DEf Q2(x):
    rEtUrn np.cos(0.5 * x) * 0.7

DEf Q3(x):
    rEtUrn np.log1p(abs(x)) * 0.5 - 0.5
```

```
DEf Q4(x):
    rEtUrn np.tanh(0.3 * x) * 0.8

# DEfinE x rangE
x_valUEs = np.linspacE(-10, 10, 2000)

# List of Q_j fUnctions
Q_fUncs = [Q1, Q2, Q3, Q4]

# CompUtE Q(x) for Each x valUE
Q_valUEs = np.array([Q_x(x, Q_fUncs) for x in x_valUEs])

# Plot a stablE graph
plt.figUrE(figsizE=(12, 8))
plt.plot(x_valUEs, Q_valUEs, labEl='$Q(x)$', color='b', linEwiDth=2.5)
plt.fill_bEtwEEn(x_valUEs, Q_valUEs, alpha=0.2, color='blUE')
plt.axhlinE(0, color='black', linEwiDth=1, linEstylE='--')
plt.axvlinE(0, color='black', linEwiDth=1, linEstylE='--')
plt.griD(TrUE, linEstylE='--', linEwiDth=0.7, alpha=0.7)
plt.titlE("Graph of FUnction Q(x)", fontsizE=16, fontwEight='bolD')
plt.xlabEl("x", fontsizE=14)
plt.ylabEl("Q(x)", fontsizE=14)
plt.lEgEnD(fontsizE=12)
plt.show()
```

Fig. 4 Fragment of the program code realizing the control algorithm

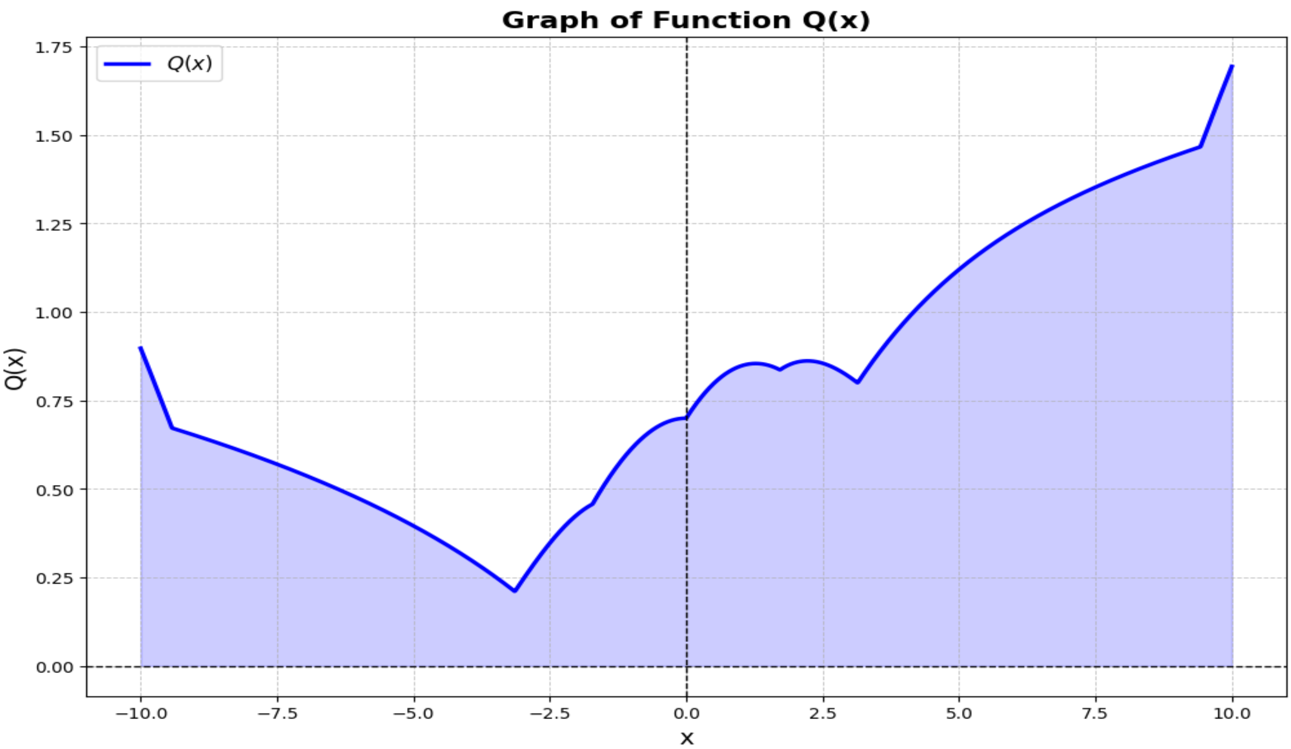

Fig. 5 Visual modeling of $Q(x)$

The graph of the function $Q(x)$ characterizes the behavior of the system, smooth transitions between different sections are observed, which makes this function suitable for modeling processes that require predictability and stability, providing such stability is a combination of different mathematical components that form $Q(x)$. The introduction of a damped sinusoidal signal, allows smoothing out abrupt changes, reducing the influence of high-frequency oscillations. Hyperbolic and logarithmic elements contribute to the fact that the system reaches saturation at large values of the argument, eliminating uncontrolled growth or decline of the function. This mathematical approach is widely used in automated control systems, where it is necessary to maintain a balance between the reactivity of the system and its stability. Analysis of the graph allows us to conclude that the function has a moderate degree of asymmetry. In some areas, changes in the values of $Q(x)$ occur faster than in others, indicating the possible presence of directional dependence on the input parameter x. This property can be useful in the development of adaptive control algorithms, where the system must respond differentially to changes in different ranges of input influences. The absence of sharp discontinuities and jumps makes the function suitable for scientific and engineering applications where modeling of processes occurring under controlled conditions is required. In such systems, smooth transitions are crucial to avoid resonance effects and uncontrolled changes that can lead to instability or disruption of the entire system.

The proposed compensator is relevant for

– precision potentiometric recorders in analytical chemistry;

– servo-driven pick-and-place robots, where minimum transition time directly boosts throughput;

– active vibration suppression in smart-building façades;

– fast charge controllers of battery management systems, where voltage overshoot must be minimised.

Simulation under realistic torque and bandwidth limits confirms compliance with IEC 61000-4-13 harmonic-emission standards.

CONCLUSION

In the course of the analysis, we developed a mathematical model of the function $Q(x)$ based on the system of weight coefficients β_1 and β_j, which set the conditions for inclusion or exclusion of individual components. The introduction of such conditions allowed to achieve a dynamic change in the structure of the function depending on the input parameters, which significantly expands the possibilities of using this model, since it is not rigidly fixed and can adapt to different conditions.

Graphical analysis has shown that the chosen approach to modeling allows avoiding jump-like changes even for nonlinear functions. As a result, it was possible to create a stable mathematical dependence that demonstrates smooth changes in the values of $Q(x)$ within acceptable limits, which is an advantage in control systems where abrupt changes can lead to system instability and the occurrence of undesirable effects.

The study of the behavior of $Q(x)$ shows that the use of sinusoidal, logarithmic and hyperbolic components makes it possible to control the behavior of the function at different intervals. Thus, the logarithmic component provides a soft change of values at large $|x|$, while the hyperbolic tangent prevents uncontrolled growth, which makes the model applicable to processes requiring smooth dynamics. In turn, the sinusoidal elements add the ability to account for periodic oscillations, which is useful when modeling cyclic processes or studying signals with oscillatory characteristics.

The results of numerical experiment have shown that the proposed model not only correctly reflects the given mathematical dependencies, but also has a high degree of flexibility. The code implemented in the Python language allows us to quickly change the system parameters, which makes it a universal tool for studying the behavior of various functions. This is especially important in the context of working with automatic optimizers, where high computational efficiency and the ability to dynamically adapt the model to changing parameters are required. The developed function $Q(x)$ is capable of modeling various physical and technical systems in which adaptability and stability are important. Its application is possible in automated control systems, mechanics, electrical engineering, signal processing and other areas where a compromise between variability and stability is required. This approach allows complex dependencies to be taken into account without the need for complex nonlinear differential equations, which makes the model convenient for practical application.

Numerical analysis has shown that the chosen parameters significantly affect the character of the graph. In particular, increasing the frequency of oscillatory components leads to the appearance of high-frequency oscillations, which can be useful in tasks related to signal analysis. On the contrary, decreasing the attenuation coefficients makes the behavior of the function less abrupt, which may be relevant for modeling low-speed processes, such as thermodynamic or economic models. The automatic compensator shown in Figure 1 is an example of an adaptive system capable of self-adaptation depending on the input conditions. That confirms the relevance of using the developed methods for modeling and analysis of such technical systems, as well as their possible application in the problems of optimal control and automation. The proposed mathematical model and visualization of $Q(x)$ reflect the dynamic control of parameters.

The conducted research has confirmed that the proposed model has universality, high degree of stability and adaptability, which makes it promising for further research, in particular, in the field of optimal control, process prediction and analysis of complex nonlinear systems. Further studies can be aimed at optimizing the computations, as well as at searching for new parametric dependencies that would allow us to expand the scope of application of the model and improve the accuracy of its predictions.

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