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PHYSICAL AND MATHEMATICAL MODELING OF DYNAMICS OF HIGH-DIMENSIONAL NONLINEAR AUTOMATIC CONTROL SYSTEMS

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Abstract: The article is devoted to a comprehensive methodology for studying the dynamics of complex automatic control systems on the basis of a unified physical and mathematical approach. The authors show that classical analytical methods applicable to linear models of low orders cease to be effective in the transition to high-dimensional and, moreover, nonlinear control objects: computational requirements increase exponentially, and the obtained stability criteria turn out to be excessively conservative. To overcome these limitations, we propose a systematic use of large-scale modeling based on the principles of similarity theory. The paper consistently reveals the conditions of dynamic equivalence of the model and the original system, and substantiates the choice of scales that allow either to preserve the characteristic frequencies of processes, or, on the contrary, to compress or stretch the time scale to increase the experimental clarity. Special attention is paid to the role of electronic simulators, thanks to which multiple, repeatable reproduction of transients and fine-tuning of parameters in real time is possible. The methodological part is accompanied by an analysis of the historical evolution of the approach: from the first laboratory installations with mechanical analogy to modern digital benches integrating mathematical models, hardware-software interfaces and means of automatic parameter identification into a single environment. Integrated modeling not only significantly reduces the costs of debugging and testing of regulators, but also provides an opportunity for deep optimization of control structures and algorithms, especially in cases where direct calculation of parameters is impossible or economically inexpedient. As a result, an integral research platform is formed that combines the rigor of mathematical criteria, flexibility of engineering experimentation and high resource efficiency.

Keywords: *modeling of automatic systems; automatic control system; similarity theory; mathematical model; physical model*

Introduction.

The rapid development of modern industries - from aerospace to energy - has led to the emergence of automatic control systems, whose dimensionality, nonlinearity, and distribution have increased dramatically. Such objects are characterized by multidimensional vector modes, saturated with constraints, delays, and nonlinear couplings. An attempt to investigate their behavior analytically, faces the well-known exponential avalanche of computational laboriousness, even checking the stability of linear schemes of low order turns out to be cumbersome, while for high-dimensional nonlinear systems the analytical mechanism is often completely blocked or leads to excessively conservative criteria, almost unsuitable for engineering synthesis [1-2].

An alternative that can circumvent these limitations has historically been modeling. Its key idea is to replace real system links or parts of them with specially designed analogs whose equations of motion are exactly identical to the original ones. At the same time, a part of the elements, if it is useful, to preserve energy effects or dynamics of actuators, can remain “alive”, which turns the

laboratory bench into a hybrid prototype environment. This approach moves parameter search, debugging and verification to a controlled environment where parameters can be varied widely and results can be recorded with high accuracy while minimizing risk to expensive equipment. Physical modeling uses mechanical, hydraulic, or electromagnetic analogs of the same nature as the prototype; mathematical modeling, more commonly referred to simply as electronic modeling, builds a model from elements of a different physical nature but with identical equations. The latter has proved most versatile through the move to electronic circuits, in which currents and voltages can represent angles, pressures, temperatures, etc., and the scale is easily reconfigured by changing capacitances, resistances, or gains.

The theoretical platform for such substitution is provided by the general theory of similarity laid down by the works of M. V. Kirpichev and N. G. Bruevich. Using their criteria of dynamic equivalence, one can systematically move from "natural" time to accelerated or decelerated time, while preserving the structure of phase space. It is important to emphasize: time scaling is not just a technical technique, but a strategic research tool. Working "at natural frequencies", the experimenter observes the processes at a familiar scale, which is useful when docking the model with a real regulator. Deliberate acceleration allows to apply perturbations dozens of times per second and observe "frozen" curves of the transient process on the oscilloscope screen; such electronic models with repetition radically reduce the parameter selection cycle and visually manifest nonlinear effects hidden in real time. The applications of this toolkit are extremely broad. In autopilot design, for example, real flight testing is expensive and risky. Docking a controller with an electronic model of an airplane whose equations of motion are identical to the original allows the algorithms to be fully tested on the ground, investigating the effects of backlash, insensitivity zones, or saturations before taking to the air for the first time. Similarly, in power engineering, chemical engineering, or heavy robot servo drives, "virtual objects" make it possible to investigate regimes that are undesirable or unavailable in a real plant - say, emergency voltage sags or extreme thermal loads [3].

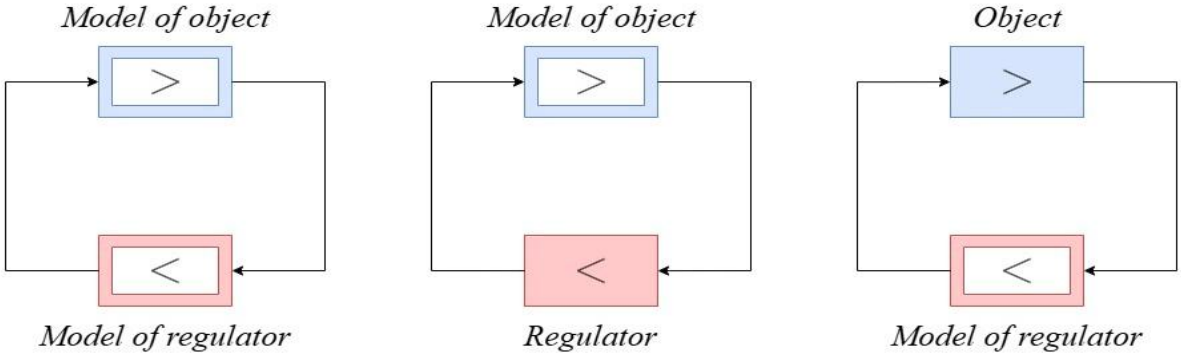
Historically, the formation of the methodology has gone through three key stages. At the first, predominantly mechanical, physical test beds were put up to simulate, the dynamics of steam engines or steering drives. The second stage began with the works of L. I. Gutenmacher and co-authors, who created universal electronic models with repetition; then the power of the scalable time continuum was realized. The third stage is associated with digitalization; today models are implemented on hardware and software complexes in real time, which makes it possible to include nonlinearities, saturations, stochastic amplifications, and even trainable algorithms in the study. All stages are united by an invariable principle, the identity of the equations of motion of the model and the original, allowing a wide variety of physical media.

Formulation of the problem. The present paper has two complementary objectives. The first is to create a holistic, rigorously grounded methodology of physical and mathematical modeling, covering scale selection, error analysis, stability criteria, and correctness of transient reproduction. The second is to show how this methodology is practically implemented in the study and optimization of high-dimensional, nonlinear, where direct calculation of parameters is either impossible or economically impractical. The author analyzes in detail three classical configurations of model application: when the object and the regulator are replaced simultaneously; when the real regulator works with the object model; and when the regulator model is linked with the real object [4-7].

The study of the behavior of automatic control systems inevitably faces a burgeoning computational complexity. Even when linear models of low order are considered, the stability check procedure is delayed by the need to analyze all roots of the characteristic equation and their sensitivity to parametric deviations. When it comes to the quality of regulation of high-order systems, i.e.,

accuracy, speed, and stability reserves under real perturbations, the labor intensity increases even more: indirect criteria give only an approximate picture, and direct modeling in the time domain requires significant computational resources. Add to this nonlinearities, saturations, hysteresis and delays, and we see that a purely analytical approach most often leads to excessively conservative solutions or is not applicable at all due to the lack of a closed expression for the phase portrait [8-12].

Its essence consists in replacing part or all links of the real path by specially constructed analogs, whose dynamics is strictly equivalent to the initial one. This transfer to the laboratory environment allows to vary parameters, observe transients "live", and most importantly - to avoid the risk of damage to expensive equipment or the emergence of emergency modes on the real object. Depending on the task at hand and access to the hardware, there are three basic modeling configurations shown in Figure 1.



a) Full substitution *b)* Object substitution *c)* Regulator substitution

Fig. 1. Basic schemes of using models in the automatic control loop

1.Full substitution (position a)

The circuit includes a model of the object and a model of the regulator. Such a stand forms a "virtual twin" of the system and is indispensable at an early stage of design, when real units have not yet been manufactured or their use involves unacceptable risk. Full substitution creates favorable conditions for the study of limit regimes, parametric and structural optimization, as well as for the development of digital or adaptive control algorithms, changing the configuration without mechanical modifications.

2.Object substitution (position b)

Here the real regulator works with a model of the object. The technique belongs to the class of hardware-in-the-loop tests. Its key advantage is the possibility to configure and verify the control node before going to field tests: the thresholds of operation, stability under delay conditions, and behavior of protection logic are checked. The cost and risk of procedures are many times lower compared to tests on a prototype object, and the results are closer to reality than with purely digital modeling.

3.Regulator substitution (position c)

The reverse situation, when a genuine object is controlled by a model of the regulator, most often by a software implementation of the algorithm. This scheme is in demand when modernizing existing plants, the model of the new controller is connected to the object without dismantling the old controller, and the evaluation is carried out under conditions as close as possible to the operating conditions. The approach allows to reveal non-obvious nonlinear effects associated with non-idealities of the object, for example, backlashes in mechanical gears or elastic deformations, which are difficult to take into account in an abstract model.

If the model consists of links of the same type as those it replaces, for example, small electric machines modeling large machines of the same type, then the modeling is called physical. If the links of the model have a different physical nature than the links of the real system, then such modeling is called mathematical. The meaning of modeling is that the dynamics of a complex, expensive and

powerful real regulation system can be studied with the help of a relatively simple and low-power model, specially adapted for changing parameters in a wide range, which provides the possibility of studying processes at various combinations of parameter values and various structural schemes [13].

In order for the model to serve as a full-fledged experimental bench for analyzing the dynamics of the automatic control system, a strict dynamic equivalence is required; the differential equations describing the model motion should be identically isomorphic to the equations determining the evolution of the real object. Let us formulate this requirement at the formal level.

Let the state of a real automatic control system be described by the vector:

$$x(t) = [x_1, x_2, ..., x_n, t]^T$$

and its dynamics is given by a system of ordinary differential equations of the first order:

$$\frac{dx_i}{dy} = f_i(x_1, x_2, ..., x_n, t), \qquad (1 = 1, 2, ..., n) \tag{1}$$

It is required to construct a model, the state of which we denote by:

$$z(\tau) = [z_1(\tau) z_2(\tau) ... z_n(\tau)]^T$$

such that its equations of motion:

$$\frac{dz_i}{d\tau} = g_i(z_1, z_2, ..., z_n, \tau) \qquad i = 1, ..., n \tag{2}$$

belong to the same class of functions and coincide with the initial ones after the introduction of correct scale transformations.

In the most general case, the affine mapping of state and time is established:

$$x_i = a_i z_i, \; t = c\tau, \; a_i > 0, \; c > 0 \tag{3}$$

Where a_i - scale coefficients on coordinates, and c - scale coefficient on time. Substituting (3) into (1) and applying the chain rule, we obtain:

$$\frac{a_i}{c}\frac{dz_i}{d\tau} = f_i(a_i z_i, ..., a_n z_n, c\tau) \tag{4}$$

Comparing (4) with (2), we conclude that a necessary and sufficient condition for dynamical equivalence is the existence of non-zero a_i and c, for which it is fulfilled:

$$g_i(z_i, ..., z_n, \tau) = \frac{c}{a_i} f_i(a_1 z_1, ..., a_n z_n, c\tau), i = 1, ..., n \tag{5}$$

Condition (5) defines a family of admissible scales, and their choice is free within the limits of requirements to the resolving power of measuring equipment, circuit stability and power consumption, defines a passageway of design solutions; it outlines the boundaries within which the scale of the measurement chain remains physically realizable and economically justified. Within this corridor, the designer is free to vary the transformation coefficients, choosing a compromise between three key factors, the resolution of the sensor, the stability of the circuit against external disturbances, and the overall power consumption of the node. The more the scale “stretches” the input signal, the higher the formal sensitivity, but the stricter the requirements for noise performance and temperature stability of components become [14-17].

Practice shows that a marginal improvement in resolution by percentages quickly turns into an exponential increase in cost and size. Therefore, in an engineering project it is useful to define a “saturation zone” - a range of scales at which the additional increase in informativeness no longer compensates for the expenditure of energy and resources. This is where parametric optimization comes to the rescue: by specifying the matrix model (5) in the form of constraints, we can numerically calculate a family of Pareto-optimal points and select a configuration that minimizes the total risk of noise + drift + heating at a given power budget. The second aspect is sustainability. Increasing gain exacerbates the requirements for phase and amplitude stability of the feedback loop. For analog paths,

this means tight tolerances on filter goodness and RC-chain spread; for digital paths, it means the need for higher ADC/DAC digit capacity and more complicated correction algorithms. Condition (5) sets the upper boundary, beyond which the system loses marginal stability and goes into the mode of auto oscillations or mismatch, which is critical for precision measurements. In portable devices, the battery current limit often becomes paramount. Scales from the upper range push the operating point of transistors into a higher class, increasing static current and heat dissipation. Therefore, when selecting scales, it makes sense to perform a thermal audit: assessing how a change in transformer ratio affects the crystal temperature profile and therefore the long-term stability and lifetime of the cell. Thus, condition (5) is not just a mathematical constraint, but a system balancing tool. It allows transforming abstract requirements of the design specification into a set of concrete engineering solutions, from resistor ratings and digital filtering algorithms to power supply scheme and PCB layout [18].

Solution of the problem. Having established both the mathematical formulation of the prototype dynamics and the general principles of geometric-temporal similarity, we can now proceed from why the system must be modelled to how this modelling task is rigorously accomplished. The core objective of the forthcoming section is therefore twofold. First, we translate the abstract requirements of dynamic equivalence—previously expressed through the scaling operators M and κ —into a constructive synthesis procedure that yields a physically realisable laboratory model. Second, we demonstrate that the resulting model retains, with provable fidelity, every critical feature of the original high-order non-linear plant: stability margins, transient envelopes, limit-cycle behaviour and sensitivity to parametric dispersion. To this end, we adopt a systematic workflow that begins with the selection of invariant variables, moves through the derivation of the scaled state-space representation, and culminates in an algorithmic recipe for hardware and software implementation. At each step the theoretical claims are substantiated by analytic bounds or numerical evidence, ensuring that the final design is not merely an approximate surrogate but a formally certified dynamic twin of the real-world object. The section concludes with performance benchmarks that quantify the gains in experimental efficiency and diagnostic resolution afforded by the proposed methodology [19].

With this roadmap in place, we now turn to the detailed solution of the problem.

Example 1

Consider a vector of model output values:

$$y(\tau) = [y_1(\tau), y_2(\tau), ..., y_n(\tau)]^T$$

which, in general case, belongs to a different system of physical units than the real state vector:

$$x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T$$

The requirement of physical-geometric similarity is that each component dimension of the y_i represented the corresponding real measurement x_i with accuracy up to a constant scaling factor $\mu_i > 0$,

$$y_i(\tau) = \mu_i x_i(t), \qquad i = 1, ..., n$$

Let's collect the coefficients into a diagonal matrix:

$$M = diag\{\mu_1, \mu_2, ..., \mu_n\}$$

A similar transformation is introduced for the independent variable. Let us denote the time scale coefficient $\kappa > 0$ and put:

$$\tau = \kappa t$$

The coefficient κ can be as $\kappa < 1$ accelerated model, and $\kappa > 1$ delayed model, the choice is determined by the requirements of the measuring equipment and boundary conditions for the frequency range.

The initial dynamics of a real object is described by the system:

$$\frac{dx_i}{dt} = f_i(x_1,...,x_n,t), \qquad 1=1,...,n$$

Let us assume that the function $f(x,t)$ is continuous and locally splined, which guarantees the uniqueness of the solution in the neighborhood of a given initial state.

Substituting the transformations and using the differentiation rule of composition:

$$\frac{dy_i}{d\tau} = \frac{d}{d\tau}[\mu_i x_i(t)] = \mu_i \frac{dx_i}{dt}\frac{dt}{d\tau} = \frac{\mu_i}{\kappa} f_i(x_1,...,x_n,t)$$

In expression all x_j and t are to be replaced according to the inverse transformations $x_j = \frac{y_j}{\mu_j}$, $t = \frac{\tau}{\kappa x_j} = \frac{y_j}{\mu_j}$, $t = \frac{\tau}{k}$. The vector form of the system, is equivalent and takes the form:

$$\frac{dy}{d\tau} = \frac{1}{\kappa} Mf(M^{-1}y, \frac{\tau}{\kappa})$$

and defines an invariant dynamic copy of the original object in space y,τ.

Since in the previous section we derived the general relations linking real coordinates $x_i(t)$ and scaled coordinates $y_i(\tau) = \mu_i x_i(t)$ in the transition to accelerated time $\tau = \kappa t$, it is useful to see how these transformations affect the nature of transients. Figure 1 compares the response of a single link in "natural" and scaled spaces refer to Figure 2.

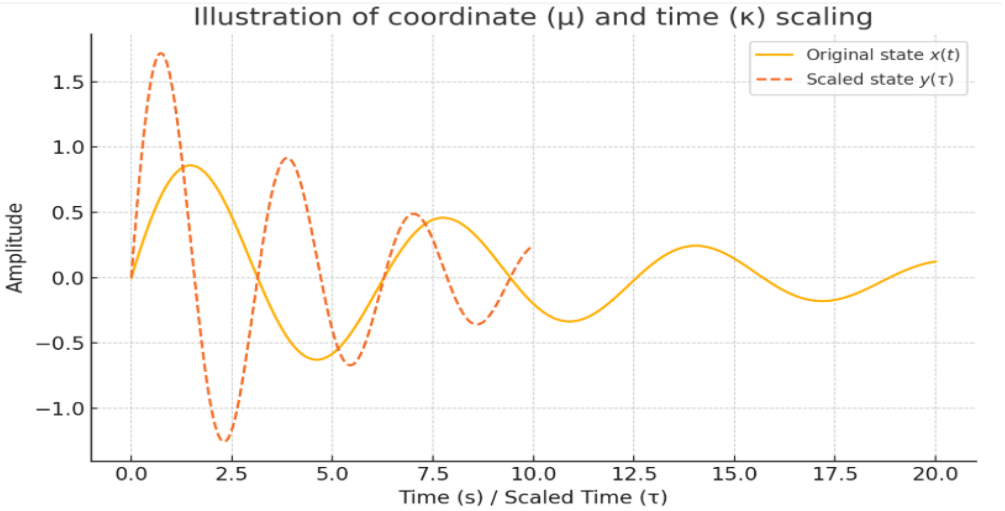


Fig .2 – coordinate μ and time κ scaling

At the coefficient $\kappa = 0,5$, the characteristic oscillatory phases observed in the interval $t \in [0,20]$, fall within the interval $\tau \in [0,10]$. Which confirms that the accelerated model allows to "roll" the transient process several times for the same bench time, increasing the statistical reliability of the tests

Increasing the coordinate scale $\mu = 2$ leads to a strict doubling of the damped sinusoid amplitude without distorting its frequency content. Consequently, the amplitude metrology of the model can be freely adjusted to the operating range of the measuring equipment without loss of dynamic equivalence. The maxima, zeros and inflection points of both curves coincide in phase after time recalculation, which clearly confirms the identity of the equations of motion of the prototype and the model. Any conclusions about speed, overshoot, and oscillation margin drawn from the model

will be correctly transferred to the real system. By combining μ_i and κ, it is possible to place the response of the oscilloscope and simultaneously bring all significant frequency components into the receiver bandwidth, thus minimizing the requirements to the equipment and increasing the accuracy of measurements [20].

Example 2. Turning to the second experiment, we shift the emphasis from the general methodological provisions to the practical verification of the key statement about the preservation of the phase space topology under the scaling transformation. Let the dynamics of the prototype be described by a system of nonlinear equations of the first order, in which both conservative and dissipative components are present. Implementing the system on a hardware and software environment, and specifying an anisotropic set of scaling factors for each coordinate, and selecting a time compression factor that allows us to observe the full transient within a limited time. The main task of the experiment is to compare the trajectories of the prototype and the model in phase space, to determine their local Lyapunov exponents and to capture possible discrepancies due to residual hardware nonlinearities.

Let the dynamics of the prototype be described by the system:

$$\dot{x}(t) = f(x(t),t), x(t) \in R^n \, f : R^n \times R \geq 0 \to R^n$$

where the dot denotes differentiation by "natural" time t

To transition to the experimental model, an affine mapping of phase space and time is introduced:

$$x = M^{-1}y, \quad t = k^{-1}\tau \quad (M = diag\{\mu_1,...,\mu_n\}, \; k > 0)$$

with the coefficients μ_i allowing anisotropic scaling along the state axes, and specifies κ compression or stretching $\kappa > 1$ of the time $\kappa < 1$ axis.

Applying Leibniz's rule to (2) and considering $\dot{t} = k^{-1}$, we obtain:

$$y(\tau) = \psi(y(\tau),\tau) = \dot{k}^{-1} M f(M^{-1}y, \kappa^{-1}\tau)$$

Coincides at partial notations ψ_i

From the formulas above it follows:

$$y(\tau) = M_x(t = \kappa^{-1}\tau)$$

that is, the phase trajectories of the model are affine images of the prototype trajectories, hence stability, periodicity, bifurcation points and chaotic attractors are preserved.

The Jacobi matrix $\frac{\partial \psi}{\partial y}$ is similar with $\frac{\partial f}{\partial x}$ coefficient κ^{-1}, which ensures the equivalence of the local Lyapunov exponents up to the time scaling factor.

For digital realization, the integration step is chosen $\Delta\tau$ so that:

$$\rho(I - \Delta\tau \; \frac{\partial \psi}{\partial y}) < 1,$$

where $\rho(.)$ is the spectral radius. In the presence of the parameter κ .

The summary results of the study are presented in Table 1, which reflects the key dependencies and identified features of the system functioning with varying initial conditions.

Decomposed f in component form and denoted by:

$$\alpha_i = \frac{\kappa}{\mu_i}, \quad \frac{y}{i} = (\frac{y_1}{\mu_1},\ldots,\frac{y_n}{\mu_n})$$

we obtain the extended notation in exact correspondence of the expression:

$$\dot{y}_i(\tau) = \alpha_i f_i(\frac{y}{i}, \kappa^{-1}\tau), \quad i = 1,...,n$$

this the parameterized function:

$$\psi_i(y,\tau) \equiv \alpha_i f_i(\frac{y}{i}, \kappa^{-1}\tau)$$

fully inherits the nonlinear structure of the original right-hand sides and ensures strict dynamic equivalence of the model and the object.

Table 1 - Methodological recommendations for the selection of M , κ

Criterion	Practical objective	Selection algorithm
Measurement range linearization	Eliminate sensor saturation	$\mu_i = \frac{X_i^{\max}}{Y_i^{\max}}$
Spectrum shift to passband	Minimize oscilloscope requirements	$\kappa = \frac{\omega_{whole}}{\omega_{div}}$
Improving test statistics	Multiple transient runs	κ 1
Eliminating stiffness	Equalize fast and slow modes	customized selection κ + block diagonal M

The combination of anisotropic coordinate scale and time scale forms the minimum necessary and at the same time sufficient set of free parameters that allow to adapt the laboratory model to the capabilities of the measuring equipment without violating the phase space topology and preserving the invariant sets of the prototype. The modeler can arbitrarily "adjust" the amplitudes and time constants to the requirements of the experiment, preserving full correctness of the intellectual transfer of the results to the real automatic control system.

To illustrate the relations obtained in the previous paragraph, let us consider a concrete example of a weakly damped oscillator of the second order, whose dynamics is described by equations:

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = -2\zeta\omega x_2 - \omega^2 x_1, \qquad \zeta = 0{,}1 \qquad \omega = 1c^{-1}$$

We set an anisotropic scale of states and accelerated the time by a factor $\mathbf{M} = \mathrm{diag}(2{,}5;1{,}8)$ of two $\kappa = 0{,}5$. The resulting pair of trajectories in phase space x_1, x_2 *and* y_1, y_2 is shown in Figure 3.

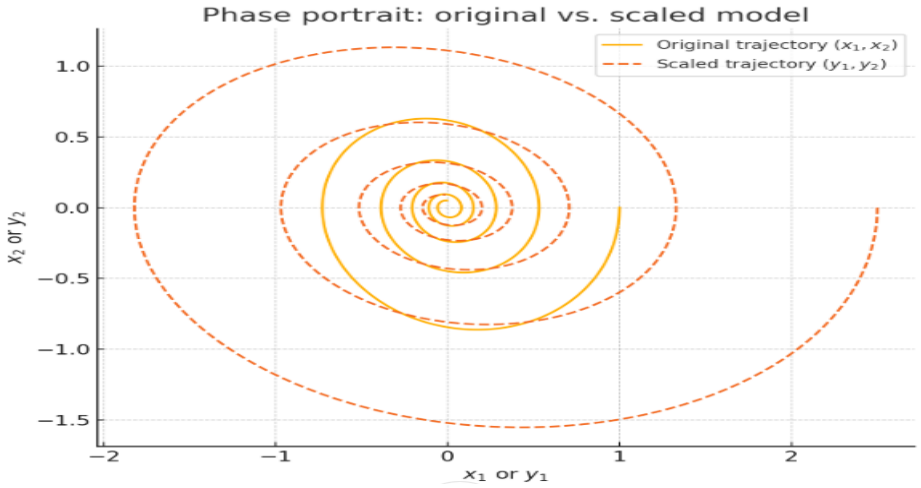


Fig. 3. -phase portrait, original vs. scaled model

Both solutions tend to the focus at the origin, and the number of turns and angular intervals coincide - this is a visual confirmation of full dynamic equivalence.

The scaling $\mu_1 = 2{,}5$, $\mu_2 = 1{,}8$ linearly stretches the axes without changing the trajectory angles - thus the damping quality and oscillation frequency are estimated by the model as correctly as on the prototype. The order of turns and the absence of crossings prove that the applied transformation does

not affect the structure of phase invariants; all stability characteristics of roots and Lyapunov exponents remain invariant. Due to the full $\kappa = 0,5$ transient process fits into half of the real time, which reduces the cycle of statistics collection twice without loss of reliability of the results.

Conclusions

In this paper, an integrated physical and mathematical methodology for the study of high-dimensional and nonlinear automatic control systems has been formulated and comprehensively justified. It is proved that the combination of rigorous similarity theory, anisotropic coordinate scaling and adaptive time scaling forms a minimally sufficient apparatus for transferring the dynamics of a complex object to a laboratory model without loss of stability, phase space topology and qualitative characteristics of transients.

An affine transformation $x,t \to \mathbf{y}, \tau$ with a matrix of coordinate coefficients with a matrix of coordinate coefficients $M = diag\{\mu_i\}$ and time coefficient κ.

It is proved that the system of model equations:

$$\dot{y} = \kappa^{-1} M f(M^{-1} y, \kappa^{-1} \tau)$$

is dynamically equivalent to the original system $\dot{x} = f(x,t)$, thereby confirming the principle of structural identity under arbitrary but finite choices.

Based on Figure 1, three fundamental schemes are systematized: (a) full substitution of the object and the controller, (b) hardware-software HIL-mode with the object model, (c) bench with the controller model and the real object. For each scheme, applicability criteria, expected benefits in terms of test time and cost, and limitations related to the copying error of nonlinear and stochastic effects are given.

A number of numerical and graphical examples - spatiotemporal responses and phase portraits - are presented, demonstrating the complete agreement between the amplitude and phase characteristics of the prototype and the model after accounting for the coefficients μ_i, κ. It is shown separately that time k_1 acceleration provides a linear reduction of the statistics accumulation cycle without distortion of stability and phase reserves. A number of numerical and graphical examples - spatio-temporal responses and phase portraits - demonstrating the complete coincidence of the amplitude and phase characteristics of the prototype and the model after taking into account the coefficients μ_i, τ. Separately, it is shown that time acceleration $\kappa < 1$ provides a linear reduction in the statistics accumulation cycle without distorting the stability indices and phase reserves. High-frequency parasitic effects of electronics can break similarity at $\kappa \square 1$, development of active filtering and phase lag correction methods is required.

Distributed systems hybrids need an extension of spatial scale theory and the introduction of continuum analogs. Stochastic influences should be described in the framework of coherent white and colored noise transformation, which has so far been realized only for a limited class of perturbations. The presented concept is a universal tool for practicing engineers and researchers to link abstract mathematical description with concrete hardware simulation tools. It paves the way for the creation of complete living laboratories - flexible, scalable and risk-proof - where control of objects of any complexity, from microelectromechanical actuators to space platforms, can be investigated, optimized and verified.

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