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ON THE MATRIX GENERALIZATION OF THE THEORY OF MACHINING ACCURACY

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Abstract: The article provides a matrix generalization of distortion models and scattering fields of dimensions performed during turning with a spatial arrangement of the tool, taking into account the simultaneous action of all components of the cutting forces of the setting tool and elastic deformations of the technological system in all coordinate directions. A full-factor model of dimension distortion for single-carriage adjustment has been developed, which allows taking into account not only plane-parallel movements of technological subsystems, but also their angular movements around base points. Thus, the Eq. developed for the total displacement vectors $\overline{u_i}$ is proposed to be taken as the basis for a full-factor model of the machining error. The presented analytical models describe only plane-parallel displacements of contacting bodies. It is shown that in order to take into account the whole complex of displacements in them, i.e. and angular displacements, it is sufficient to replace the plane-parallel displacement vectors of each contacting body $\overline{r_i}$.

Keywords: *matrix generalization of the theory of accuracy, turning, distortion models, models of scattering fields, performed dimensions, coordinate displacements of the technological system, compliance matrix of technological system, machining error*

Introduction. Machining accuracy is predetermined by a whole complex of random and regular factors, their mutual influence and interaction: dimensional wear of the cutting tool (Δ_w), temperature deformations ($\Sigma\Delta_t$) and geometric inaccuracies of the technological system links ($\Sigma\Delta_m$), errors in the installation of workpieces on the machine ($\Delta\varepsilon_i$) and its settings for the size being performed (Δ_s), spread of allowances and physical and mechanical properties of workpieces, etc. [1, 2]. The machining accuracy has its numerical expression through the machining error, which characterizes the degree of discrepancy between the real part and the ideal scheme underlying the machining method. All elementary error components can be divided into two groups:

- independent or weakly dependent on cutting conditions: Δ_s, Δ_w, $\Delta\varepsilon_i$, $\Sigma\Delta_m$, $\Sigma\Delta_t$;
- determined by cutting conditions: Δy.

The first group of errors is not of interest in the development of a simulation model intended for designing a technological process. These components in the simulation model participate as constants, the values of which are taken from the extensive reference literature [2].

A special place in modeling the accuracy of machining is occupied by the elementary error Δy, which occurs due to the elastic displacements of the technological system under the action of cutting forces. Its value is directly determined by the cutting conditions and the characteristics of the technological system. Therefore, it is the main control object, which requires a strict mathematical description.

The foundations of accuracy modeling were laid by A.P. Sokolovsky and K.V. Votinov [3]. Further study of these issues was carried out by B.S. Balakshin, V.S. Korsakov, B.M. Bazrov [4].

Since the machining error is characterized by a number of different indicators, two groups of machining error models can be distinguished:

- dimension distortion model;
- model of scattering field of dimension.

For the simplest case of turning, the scheme for the formation of dimension distortion was formed by V.S. Korsakov (Fig. 1.).

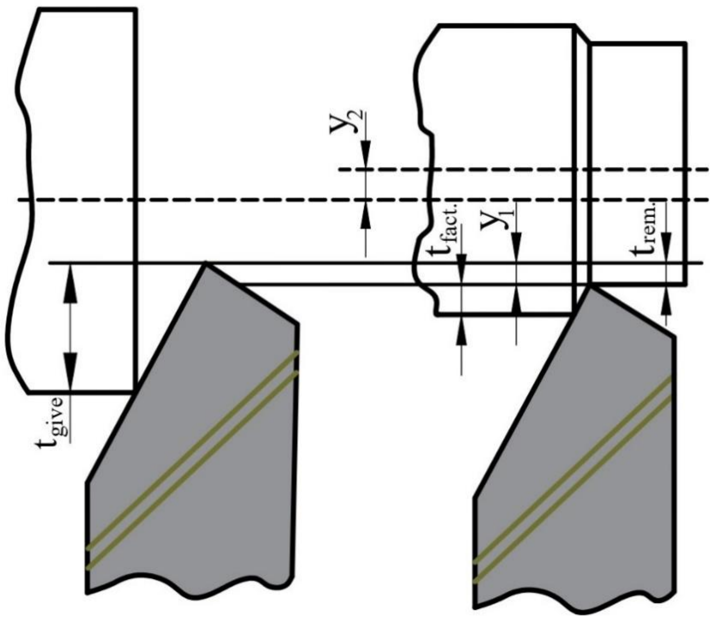

Fig.1. Scheme of elastic displacements of the technological system during turning [3]

Under the action of cutting forces, the links of the technological system are displaced from the initial (unloaded) state, which ultimately causes a violation of the relative position of cutting edge of the tool and the workpiece surface established by the adjustment. Since the technological system in the range of loads inherent in the cutting process is a linear elastically deformable system, he proposed a formula for the value of the mutual displacement of the tool and workpiece: $y = y_1 + y_2$, which makes it possible to calculate the dimension distortion in the diametrical direction depending on the machining conditions.

When machining a batch of workpieces, the cutting force changes as a result of uneven depth of cut (due to the variability of the size of the workpieces in the batch) and the instability of the mechanical properties of the material of the workpieces. The instability of the cutting force leads to the inconstancy of the amount of elastic compression, and, consequently, the size of the part in the batch. B.S. Balakshin formulated the principle of taking into account fluctuations in the allowance when calculating the machining error [3].

V.S. Korsakov proposed analytical dependencies to determine the error of the performed dimension of single-tool turning. He proposed to determine the scattering field of the dimension being performed in a given section as the difference between the largest and smallest values of the residual (not removed due to elastic displacements) depth of cut. However, he takes into account only fluctuations in the allowance and hardness of the material being machined, and these are not all the factors that determine the scattering of elastic deformations.

K.V. Votinov introduced into the scattering field model, along with the traditional component, determined by the allowance fluctuations, a new one, determined by the depth of cut. However, the coefficients in this formula are determined experimentally, and therefore the entire dependence is empirical.

Formulation of the problem. The component of the machining error that occurs due to the elastic displacement of the elements of the technological system under the influence of cutting forces, which is often called the deformation component, is the most controllable during the machining process and at the design stage. By varying the cutting conditions, the geometry of the cutting tool, the cutting material, the initial error, one can significantly influence the magnitude of the machining error. Therefore, the mathematical model of the deformation component of the machining error is *the basis of the computational theory of machining accuracy*.

It is generally accepted that the model of the deformation component of the machining error can be built using the theory of linear deformable systems [5]. However, when describing the properties of a deformable system, in our case it is a technological system, the question arises about the degree of detail of the description. The L.P. Medvedev model takes into account one characteristic of the system - its total rigidity, while the B.M. Bazrov model takes into account the rigidity of all elements of the system [3, 5]. The second approach is absolutely correct methodically and leads to a strict logical statement of the problem, however, in this case the model becomes very cumbersome and cannot be analyzed analytically. In addition, the practical determination of the rigidity of each element is associated with significant difficulties.

Therefore, it is proposed, in accordance with the methodology of B.M. Bazrov, to decompose the technological system into its component parts, but limit it to the level of subsystems, and the number of subsystems should be minimal [5]. Taking into account the specifics of automatic lathe machining (sufficient rigidity of the part, its predominant cantilever fastening), it is proposed to distinguish the following subsystems:

- spindle - chuck (collet) - part;
- carriage – holder – tool.

For each of these subsystems, it can be considered that, under the influence of cutting forces, it experiences elastic displacement as a single element. The rigidity of such a subsystem can be easily determined experimentally using a modified production method [5]. The production method makes it possible to evaluate the rigidity during machining and therefore simultaneously takes into account the dynamics of the deformation process.

The true dynamic characteristics of the rigidity of the machine tool and other elements of the technological system (amplitude-frequency and amplitude-frequency-phase characteristics) more accurately describe the resistance of deformable elements during the application and removal of the load [6]. However, their values for automatic turning equipment are not available in the reference literature, and the methods of experimental evaluation have been worked out only for the simplest cases.

Distortion of performed dimension. In accordance with the formulation of the problem, we decompose into two subsystems:

- subsystem 0: “spindle-chuck-part” (rigidity along the coordinate axes Y and X respectively j_{y0} and j_{x0});
- subsystem 1: “carriage-holder-tool” (rigidity along the coordinate axes Y and X respectively j_{y1} and j_{x1}).

Then the calculation scheme of V.S. Korsakov (Fig. 1) is converted to the form shown in Fig. 2.

Under the action of force P_{y0} subsystem 0 has displacement y_0, and subsystem 1 under the action of reaction P_{y1} has displacement y_1. Considering each subsystem as elastically deformable, we obtain:

- for displacements of subsystem 0 along the Y and X axes, respectively:

$$y_0 = \frac{P_y}{j_{yo}}, \qquad x_0 = \frac{P_x}{j_{xo}}$$

- for displacements of subsystem 1 along the Y and X axes:

$$y_1 = -\frac{P_y}{j_{y1}}, \qquad x_1 = -\frac{P_x}{j_{x1}}$$

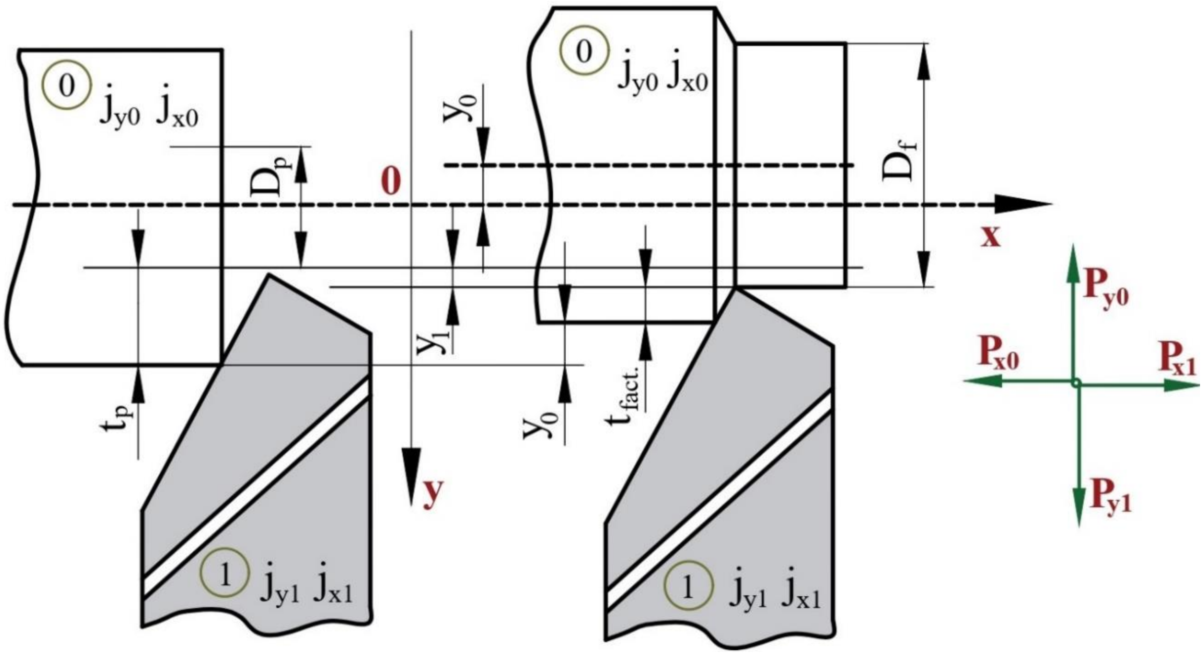

Fig. 2. Design scheme for elastic displacements of subsystems of technological system in turning

Performing the vector summation of these displacements in accordance with the design scheme, we obtain the distortions of the performed dimensions along each axis with respect to the static setting:

$$y = y_0 - y_1 = P_y\left(\frac{1}{j_{y0}} + \frac{1}{j_{y1}}\right) = \frac{P_y}{j_{y01}}; \qquad x = x_0 - x_1 = P_x\left(\frac{1}{j_{x0}} + \frac{1}{j_{x1}}\right) = \frac{P_x}{j_{x01}} \tag{1}$$

where j_{yo1} and j_{xo1} are the total rigidity of the technological system along the coordinate axes Y and X, respectively:

$$\frac{1}{j_{y01}} = \frac{1}{j_{y0}} + \frac{1}{j_{y1}}; \qquad \frac{1}{j_{x01}} = \frac{1}{j_{x0}} + \frac{1}{j_{x1}} \tag{2}$$

Using the cutting forces $P_y = C_{p_y} t^{x_{p_y}} S^{y_{p_y}},\ P_x = C_{p_x} t^{x_{p_x}} S^{y_{p_x}}$ model and denoting the actual depth of cut t_f to distort dimensions, we get:

$$y = \frac{C_{p_y} t_{tm}^{x_{p_y}} S^{y_{p_y}}}{j_{y01}}; \qquad x = \frac{C_{p_x} t_{tm}^{x_{p_y}} S^{y_{p_x}}}{j_{x01}} \tag{3}$$

As follows from the design scheme (Fig. 2), the actual depth of cut t_f is expressed through the calculated *t*:

$$t_f = t - y_0 - y_1 = t - y \tag{4}$$

Then from (3) we arrive at the system of transcendental Eqs.:

$$\begin{cases} y = \dfrac{C_{p_y}(t-y)^{x_{p_y}} S^{y_{p_y}}}{j_{y01}} \\ x = \dfrac{C_{p_x}(t-y)^{x_{p_x}} S^{y_{p_x}}}{j_{x01}} \end{cases} \tag{5}$$

Since the condition y<< t is valid for the case of cutting edge machining, it is possible to get rid of transcendence in expressions (5) by linearization:

$$(t-y)^{x_{p_y}} = t^{x_{p_y}}\left(1-\frac{y}{t}\right)^{x_{p_y}} \approx t^{x_{p_y}}\left(1-\frac{y}{t}x_{p_y}\right)$$

Solving the resulting linear Eq. with respect to y, we obtain:

$$y = \frac{1}{1+\frac{C_{p_y} t^{x_{p_y}-1} S^{p_y}}{j_{y01}}\cdot x_{p_y}} \cdot \frac{C_{p_y} t^{x_{p_y}} S^{p_y}}{j_{y01}} \qquad (6)$$

Since, under real conditions of turning, the elastic displacements of the technological system are much less than the allowance to be removed (calculated depth of cut), the influence of the correction in the denominator can be neglected (see Table 1).

Table 1. Influence of the correction in the denominator in formula (6) [5].

№	Machining conditions	$\frac{C_{p_y} t^{x_{p_y}-1} S^{y_{p_y}}}{j_{y01}} \cdot x_{p_y}$
1	External turning of structural steel with a carbide cutter on a 1K62 machine, t = 2 mm, S = 0.15 mm/rev	0,065
2	Rough turning, t = 4 mm, S = 0.6 mm/rev	0,11
3	Turning with a high speed tool, t = 2 mm	0,025

Thus, for cutting edge machining, we can take the dependence:

$$y \approx \frac{C_{p_y} t^{x_{p_y}} S^{y_{p_y}}}{j_{y01}} \quad ; \qquad x \approx \frac{C_{p_x} t^{x_{p_x}} S^{y_{p_x}}}{j_{x01}} \qquad (7)$$

The resulting expressions are close to the L.P. Medvedev model [5]. However, the total rigidity of the technological system along the coordinate axes Y and X works here, which have a clear physical meaning and, taking into account expressions (2), allow a rigorous experimental determination, for example, using a modified production method [5].

Scattering field of performed dimension. Expressions (7) make it possible to calculate the distortions of the performed dimensions, that is, to estimate the error of the static adjustment. These expressions can be used to calculate the adjustment size. However, a more relevant characteristic of the accuracy of the performed dimension is the magnitude of the scattering field.

The works of B.S. Balakshin and V.S. Korsakov laid the foundations for the calculation of the scattering field, they also identified the main factors that predetermine the occurrence of scattering fields (fluctuations in the allowance and strength properties of the material being machined in a batch of parts).

In production, the machining of a batch of parts is carried out not on one machine, but on a certain group of machines (one model), which also have a spread in their characteristics. Therefore, the stiffness value in model (7) also has a spread. According to GOST 43-85, 18097-88, 6820-75 lathes of normal accuracy have an allowable variation in rigidity of about 20% [7]. The spread of the strength properties of workpieces is also regulated: for rolled products, the tolerance for ultimate strength σ_b is *20%* [2, 5].

Therefore, the main technological factors causing the appearance of scattering fields can be called [1, 3, 5]:

- machining allowance fluctuations $\quad t \in \left[t-\frac{\Delta t}{2};\ t+\frac{\Delta t}{2}\right] \qquad (8)$

- variability in mechanical properties (e.g. hardness) of workpieces within a batch

$$C_{tm} \in C, \left[1-\frac{\nu}{2};\ \ 1+\frac{\nu}{2}\right] \tag{9}$$

- variation in rigidity of different machines of the same model

$$j_{tm} \in j, \left[1-\frac{\Delta j}{2};\ \ 1+\frac{\Delta j}{2}\right] \tag{10}$$

Such a mathematical representation of possible fluctuations in the properties of the technological system indicates the way for calculating the magnitude of the scattering field.

To determine the scattering field of the dimension being performed (its dynamic component), it is necessary to find the limiting values of the dimension distortion.

$$y_{max} = \frac{C_{p_y}\left(1+\frac{\nu}{2}\right)\left(t+\frac{\Delta t}{2}\right)^{x_{p_y}} S^{y_{p_y}}}{j_{y01}\left(1-\frac{\Delta j}{2}\right)} \tag{11}$$

$$y_{min} = \frac{C_{p_y}\left(1-\frac{\nu}{2}\right)\left(t-\frac{\Delta t}{2}\right)^{x_{p_y}} S^{y_{p_y}}}{j_{y01}\left(1+\frac{\Delta j}{2}\right)} \tag{12}$$

As a result, for the scattering field we obtain:

$$\Delta y = \frac{C_{p_y} t^{x_{p_y}} S^{y_{p_y}}}{j_{y01}} \cdot \left[\frac{\left(1+\frac{\nu}{2}\right)\left(1+\frac{\Delta t}{2t}\right)^{x_{p_y}}}{\left(1-\frac{\Delta j}{2}\right)} - \frac{\left(1-\frac{\nu}{2}\right)\left(1-\frac{\Delta t}{2t}\right)^{x_{p_y}}}{\left(1+\frac{\Delta j}{2}\right)}\right] \tag{13}$$

For the obtained dependence (13), linearization is admissible, since $0.5\nu << 1$, $0.5\Delta j << 1$ and for cutting edge machining, $\Delta t/2t < 1$ is true. Therefore, one can write:

$$\left(1-\frac{\Delta t}{2t}\right)^{x_{p_y}} \approx 1-\frac{x_{p_y}\Delta t}{2t}; \qquad \frac{1}{1-\frac{\Delta j}{2}} \approx 1+\frac{\Delta j}{2}; \qquad \frac{1}{1+\frac{\Delta j}{2}} \approx 1-\frac{\Delta j}{2}$$

After substitution into (13) we obtain:

$$\Delta y = \frac{C_{p_y} t^{x_{p_y}} S^{y_{p_y}}}{j_{y01}} \left[\left(1+\frac{\nu}{2}\right)\left(1+\frac{\Delta j}{2}\right)\left(1+x_{p_y}\frac{\Delta t}{2t}\right) - \left(1-\frac{\nu}{2}\right)\left(1-\frac{\Delta j}{2}\right)\left(1-x_{p_y}\frac{\Delta t}{2t}\right)\right]$$

Considering that ν, Δj and Δt are small values, after discarding the values of the second order of smallness and introducing the designation $\Delta j+\nu=\omega$ for the total spread of the properties of the technological system, we have for the diametrical dimension:

$$\Delta y \approx \frac{C_{p_y} t^{x_{p_y}-1} S^{y_{p_y}}}{j_{y01}} \left[\omega t + x_{p_y}\Delta t\right] \tag{14}$$

Similarly for the linear dimension:

$$\Delta x \approx \frac{C_{p_x} t^{x_{p_x}-1} S^{y_{p_x}}}{j_{x01}} \left[\omega t + x_{p_x}\Delta t\right] \tag{15}$$

As you can see, the value of the scattering field depends not only on the fluctuations of the allowance, but has a term with the value of the allowance to be removed. Thus, these formulas are an analytical representation of the K.V.Votinov model, which predicted the existence of such a relationship.

For common cutting conditions, the dependence of cutting forces on depth is very close to linear: the exponents x_{p_y} and x_{p_x} for longitudinal turning with a carbide cutter are of the order of 0.9.

Therefore, it is permissible to use models linearized in t:

$$\Delta y \approx \frac{C_{p_y} S^{y_{p_y}}}{j_{y01}} [\omega t + \Delta t] \tag{16}$$

$$\Delta \mathrm{x} \approx \frac{C_{p_x} S^{y_{p_x}}}{j_{x01}} [\omega t + \Delta t] \tag{17}$$

Matrix models. Models of distortion of the performed dimensions (7) of the scattering field (14, 15) are built under the assumption that the displacements of technological subsystems in the *y* direction are formed by the component of the cutting force p_y, and in the *x* direction - only p_x. This assumption is valid for rigid parts of small dimensions and with a ratio of overall dimensions L = D. In general cases, all components of the cutting force affect the coordinate displacements of technological subsystems. That is why the rigidity of the spindle and tailstock is taken into consideration, the rotation of the spindle is considered and the center of rotation is calculated [5]. These influences can be taken into account based on the general laws of analytical mechanics [3].

Since the cutting force during turning is a vector in three-dimensional space and the elastic displacements of technological subsystems under the action of this force are also described by a spatial vector, generalizing the coordinate Eqs. (7) to distort the dimensions performed, we can proceed to the vector Eq. [8-18]:

$$\bar{g} = \bar{c} \cdot \bar{p} \tag{18}$$

where g – vector of elastic displacement of the technological system; $\bar{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$ - vector of cutting force; $\bar{c} = \begin{pmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{yx} & c_{yy} & c_{yz} \\ c_{zx} & c_{zy} & c_{zz} \end{pmatrix}$ - compliance matrix of technological system.

In the coordinate Eq.s (1, 2, 3, 5), the compliance of the technological system is characterized by its reciprocal value - rigidity (along the coordinate directions j_{yo1} and j_{xo1}). In terms of rigidity, the matrix *c* can be represented as:

$$\bar{c} = \begin{pmatrix} \frac{1}{j_{xx}} & \frac{1}{j_{xy}} & \frac{1}{j_{xz}} \\ \frac{1}{j_{yx}} & \frac{1}{j_{yy}} & \frac{1}{j_{yz}} \\ \frac{1}{j_{zx}} & \frac{1}{j_{zy}} & \frac{1}{j_{zz}} \end{pmatrix} \tag{19}$$

where j_{xx} and j_{yy} correspond to the stiffness along the x (j_{x01}) and y (j_{y01}) axis in the coordinate Eq.s. In the vector Eq. of the cutting force, one can use the traditional notation for the cutting theory for the coordinate components P_x, P_y, P_z.

$$p_i = c_i t^{x_i} s^{y_i} \upsilon^{z_i} \quad (\mathrm{i = x, y, z}) \tag{20}$$

Taking into account the introduced notation (19, 20), expression (18) is a vector model of the distortion of the performed dimension. The distortion of the diametral dimension (distortion in the y direction) is described by the second component of the vector $\bar{g}$:

$$g_y = y = \frac{c_y t^{x_y} s^{y_y} v^{z_y}}{j_{yy}} + \frac{c_x t^{x_x} s^{y_x} v^{z_x}}{j_{yx}} + \frac{c_z t^{x_z} s^{y_z} v^{z_z}}{j_{yz}} \qquad (21)$$

the distortion of the linear dimension is described by the first component of the vector $\bar{g}$:

$$g_x = x = \frac{c_x t^{x_x} s^{y_x} v^{z_x}}{j_{xx}} + \frac{c_y t^{x_y} s^{y_y} v^{z_y}}{j_{xy}} + \frac{c_z t^{x_z} s^{y_z} v^{z_z}}{j_{xz}} \qquad (22)$$

Eqs. (21, 22) are a matrix generalization of coordinate Eqs. (7). The first terms in the generalized Eqs. represent the right-hand sides of Eqs. (7) and describe the direct influence of the cutting force component of the same name (force P_y on the diametral dimension - y, force P_x on the linear dimension - x). The remaining terms describe the indirect influence of other components of the cutting force.

To form a vector model of the scattering field, we consider, by analogy with expressions (11, 12), taking into account the notation (8, 9, 10), the expressions for fluctuations in the cutting force:

$$\max\ p_i = c_i\left(1+\frac{v}{2}\right)\left(t+\frac{\Delta t}{2}\right)^{x_i} s^{y_i} v^{z_i} \quad (i = x, y, z) \qquad (23)$$

$$\min\ p_i = c_i\left(1-\frac{v}{2}\right)\left(t-\frac{\Delta t}{2}\right)^{x_i} s^{y_i} v^{z_i} \quad (i = x, y, z) \qquad (24)$$

The maximum compliance of the technological system is described by the following matrix:

$$max\ \bar{c} = \begin{pmatrix} \frac{1}{j_{xx}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{xy}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{xz}\left(1-\frac{\varepsilon}{2}\right)} \\ \frac{1}{j_{yx}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{yy}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{yz}\left(1-\frac{\varepsilon}{2}\right)} \\ \frac{1}{j_{zx}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{zy}\left(1-\frac{\varepsilon}{2}\right)} & \frac{1}{j_{zz}\left(1-\frac{\varepsilon}{2}\right)} \end{pmatrix} = \frac{1}{\left(1-\frac{\varepsilon}{2}\right)} \begin{pmatrix} \frac{1}{j_{xx}} & \frac{1}{j_{xy}} & \frac{1}{j_{xz}} \\ \frac{1}{j_{yx}} & \frac{1}{j_{yy}} & \frac{1}{j_{yz}} \\ \frac{1}{j_{zx}} & \frac{1}{j_{zy}} & \frac{1}{j_{zz}} \end{pmatrix} \qquad (25)$$

For minimal compliance, the matrix is transformed to the form:

$$min\ \bar{c} = \begin{pmatrix} \frac{1}{j_{xx}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{xy}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{xz}\left(1+\frac{\varepsilon}{2}\right)} \\ \frac{1}{j_{yx}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{yy}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{yz}\left(1+\frac{\varepsilon}{2}\right)} \\ \frac{1}{j_{zx}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{zy}\left(1+\frac{\varepsilon}{2}\right)} & \frac{1}{j_{zz}\left(1+\frac{\varepsilon}{2}\right)} \end{pmatrix} = \frac{1}{\left(1+\frac{\varepsilon}{2}\right)} \begin{pmatrix} \frac{1}{j_{xx}} & \frac{1}{j_{xy}} & \frac{1}{j_{xz}} \\ \frac{1}{j_{yx}} & \frac{1}{j_{yy}} & \frac{1}{j_{yz}} \\ \frac{1}{j_{zx}} & \frac{1}{j_{zy}} & \frac{1}{j_{zz}} \end{pmatrix} \qquad (26)$$

The vector analogue of Eq. (11) - the maximum distortion of the dimension being performed - will be described, taking into account the notation (23 - 26), as follows:

$$\max \bar{g} = \frac{1}{\left(1-\frac{\varepsilon}{2}\right)} \bar{c} \begin{pmatrix} c_x\left(1+\frac{v}{2}\right)\left(t+\frac{x_x}{2}\Delta t\right) s^{y_x} v^{z_x} \\ c_y\left(1+\frac{v}{2}\right)\left(t+\frac{x_y}{2}\Delta t\right) s^{y_y} v^{z_y} \\ c_z\left(1+\frac{v}{2}\right)\left(t+\frac{x_z}{2}\Delta t\right) s^{y_z} v^{z_z} \end{pmatrix} = \frac{1+\frac{v}{2}}{1-\frac{\varepsilon}{2}} \bar{c} \left[\begin{pmatrix} c_x t s^{y_x} v^{z_x} \\ c_y t s^{y_y} v^{z_y} \\ c_z t s^{y_z} v^{z_z} \end{pmatrix} + \right.$$

$$+\frac{1}{2}\begin{pmatrix} c_x x_x \Delta t s^{y_x} v^{z_x} \\ c_y x_y \Delta t s^{y_y} v^{z_y} \\ c_z x_z \Delta t s^{y_z} v^{z_z} \end{pmatrix}\Bigg] = \frac{1+\frac{\nu}{2}}{1-\frac{\varepsilon}{2}}\bar{c}\left[\begin{pmatrix} c_x s^{y_x} v^{z_x} \\ c_y s^{y_y} v^{z_y} \\ c_z s^{y_z} v^{z_z} \end{pmatrix} + +\frac{\Delta t}{2}\begin{pmatrix} c_x x_x s^{y_x} v^{z_x} \\ c_y x_y s^{y_y} v^{z_y} \\ c_z x_z s^{y_z} v^{z_z} \end{pmatrix}\right] = \frac{1+\frac{\nu}{2}}{1-\frac{\varepsilon}{2}}\bar{c}\left[t\overline{p_t} + \frac{\Delta t}{2}\overline{p_{\Delta t}}\right] \quad (27)$$

Where vector $\overline{p_t} = \begin{pmatrix} c_x s^{y_x} v^{z_x} \\ c_y s^{y_y} v^{z_y} \\ c_z s^{y_z} v^{z_z} \end{pmatrix}$ characterizes the degree of influence of the depth of cut t, vector $\overline{p_{\Delta t}} = \begin{pmatrix} x_x c_x s^{y_x} v^{z_x} \\ x_y c_y s^{y_y} v^{z_y} \\ x_z c_z s^{y_z} v^{z_z} \end{pmatrix}$ characterizes the degree of influence of allowance fluctuations.

Similarly, for the minimum distortion of the performed dimension, one can immediately write out the vector expression:

$$min\,\overline{g} = \frac{1-\frac{\nu}{2}}{1+\frac{\varepsilon}{2}}\bar{c}\left[t\overline{p_t} - \frac{\Delta t}{2}\overline{p_{\Delta t}}\right] \quad (28)$$

By analogy with expressions (14, 15), we obtain a vector expression for the scattering field:

$$\Delta g = \frac{1+\frac{\nu}{2}}{1-\frac{\varepsilon}{2}}\bar{c}\left[t\overline{p_t} + \frac{\Delta t}{2}\overline{p_{\Delta t}}\right] - \frac{1-\frac{\nu}{2}}{1+\frac{\varepsilon}{2}}\bar{c}\left[t\overline{p_t} - \frac{\Delta t}{2}\overline{p_{\Delta t}}\right] = \bar{c}\left\{t\overline{p_t}\left(\frac{1+\frac{\nu}{2}}{1-\frac{\varepsilon}{2}} - \frac{1-\frac{\nu}{2}}{1+\frac{\varepsilon}{2}}\right) + \frac{\Delta t}{2}\overline{p_{\Delta t}}\left(\frac{1+\frac{\nu}{2}}{1-\frac{\varepsilon}{2}} + \frac{1-\frac{\nu}{2}}{1+\frac{\varepsilon}{2}}\right)\right\} =$$

$$\left|\begin{matrix} \dfrac{1+\frac{\nu}{2}+\frac{\varepsilon}{2}+\frac{\nu\varepsilon}{4}-1+\frac{\nu}{2}+\frac{\varepsilon}{2}-\frac{\nu\varepsilon}{4}}{1-\frac{\varepsilon^2}{4}} = \nu+\varepsilon = \omega \\ \dfrac{1+\frac{\nu}{2}+\frac{\varepsilon}{2}+\frac{\nu\varepsilon}{4}-1-\frac{\nu}{2}-\frac{\varepsilon}{2}-\frac{\nu\varepsilon}{4}}{1-\frac{\varepsilon^2}{4}} = 2 \end{matrix}\right| = \bar{c}\{\omega t\overline{p_t} + \Delta t\overline{p_t}\} \quad (29)$$

Expression (29) is a matrix generalization of the model of the scattering field of the performed dimensions. The transition to a specific dimension is reduced to considering the corresponding coordinate of the vector Δg. For example, for the magnitude of the scattering field of the diametrical dimension, the relation is true:

$$\Delta y = \Delta g_y \quad (30)$$

for a linear dimension we get:

$$\Delta x = \Delta g_x \quad (31)$$

Full-factor model of dimensional distortion. All models of machining error (distortions in dimensions, scattering fields of dimensions) formed in (7), (14, 15), (18), (29) take into account only plane-parallel movements of the subsystems of the technological system along the coordinate axes of the Cartesian coordinate system X, Y, Z. Such an approach to modeling the process of formation of machining errors is acceptable for parts that have overall dimensions of the same order in all coordinate directions. However, in practice, it is not uncommon for turning operations to machine parts with overall dimensions that differ significantly in different directions. For example, long shafts (predominant linear dimension), disks and flanges (predominant diametrical dimension). In these cases, a significant contribution to the machining error can be made by the rotation of the workpiece, especially in the directions of the prevailing overall dimensions.

The need to take into account the angular displacements of the workpiece under the action of cutting forces was pointed out in the works of A.P.Sokolovsky, V.S.Korsakov, D.D.Medvedev and others. [3]. They offer even the simplest analytical dependences for calculating these angular displacements. However, all these dependences are of a particular nature, they include a number of parameters, the determination of which in practice is associated with insurmountable difficulties. For example, the center of rotation of the spindle is generally a virtual object that cannot be practically measured. Most importantly, these models do not agree with the general laws of the mechanics of elastically deformable systems. Therefore, they cannot be used to build a unified theory of machining accuracy, taking into account the possible angular displacements of the subsystems of the technological system.

As is known from analytical mechanics, a body in space has 6 degrees of freedom:

-3 plane-parallel movements along the coordinate axes X, Y, Z;

- 3 rotations around each of the coordinate axes [9, 13, 15].

Fixing the position of a rigid body in space is carried out by imposing constraints on each degree of freedom. Three bonds limit the plane-parallel movement along the corresponding coordinate axes and three bonds limit the angular displacement of the body around each of the coordinate axes. The level of restriction of the freedom of movement of the body, created by the superimposed connection, is characterized by the rigidity of the connection, or its reciprocal value - the compliance of the connection.

In [9, 13, 15], to describe the displacements of a body in space, taking into account all six degrees of freedom, its position is given by two parameters (Fig. 3) [4]:

- point O (x_0; y_0; z_0) belonging to the body;

- vector $\bar{l}$ of unit length, belonging to the body and directed, for example, along its prevailing direction.

All plane-parallel displacements of the body are characterized by displacements of point O. The angular displacements of the body are described by rotations of the vector $\bar{l}$ around the point O.

The movements of the body occur as a result of the force $\bar{F}$ applied at point A (x, y, z), also belonging to the body.

Let the vector $\bar{r} = (r_x, r_y, r_z)$ be the plane-parallel displacement of the point O, the vector $\bar{\omega} = (\omega_x, \omega_y, \omega_z)$ the angle of rotation of the body, and formally the vector $\bar{l}$, which specifies the orientation of the body in space, relative to the point O. Here r_x, r_y, r_z - are movements along the coordinate axes, ω_x, ω_y, ω_z - are rotation angles around the corresponding coordinate axes.

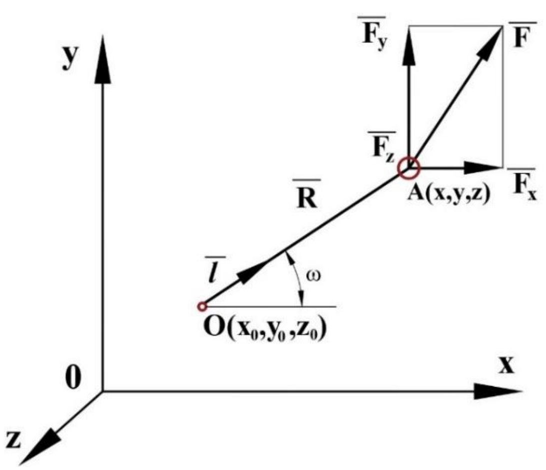

Fig. 3. Design scheme of body displacements under the action of an applied force, taking into account six degrees of freedom [4]

Then the total displacement *u* of point *A* is made up of a plane-parallel displacement and an angular displacement:

$$\bar{u} = \bar{r} + \bar{\rho} \tag{32}$$

where the second term describes exactly the angular displacements of the point *A*:

$$\bar{\rho} = \bar{\omega} \times \bar{R} \tag{33}$$

Vector $\bar{R}$ specifies the orientation of point *A* (point of application of force $\bar{F}$) relative to point *O*. It is with respect to this point that the angular displacements of point *A* are considered:

$$\bar{R} = \overline{OA} = \{x - x_0;\ y - y_0;\ z - z_0\} \tag{34}$$

The plane-parallel displacement of the point O under the influence of the force $\bar{F}$, taking into account the compliance of the superimposed bonds, is determined, in accordance with Eqs. (318), as:

$$\bar{r} = e\bar{F} \tag{35}$$

where *e* is the compliance matrix $\bar{e} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix}$

In coordinate form, Eq. (3.329) takes the form:

$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \tag{36}$$

The rotation angles given by the vector $\bar{\omega}$ are determined by the angular compliance matrix. $\xi = \begin{pmatrix} \xi_{xx} & \xi_{xy} & \xi_{xz} \\ \xi_{yx} & \xi_{yy} & \xi_{yz} \\ \xi_{zx} & \xi_{zy} & \xi_{zz} \end{pmatrix}$ The governing Eq. is constructed in the image of the Eq. for plane-parallel displacements (18), but instead of force, the moment of force $\bar{M}$ is involved in it:

$$\bar{M} = \bar{R} \times \bar{F} \tag{37}$$

Then for the rotation of the vector $\bar{l}$, which characterizes the orientation of the body in space, we obtain, by analogy with (35):

$$\bar{\omega} = \xi\bar{M} = \xi \cdot (\bar{R} \times \bar{F}) \tag{38}$$

In coordinate form, this Eq. has the form:

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} \xi_{xx} & \xi_{xy} & \xi_{xz} \\ \xi_{yx} & \xi_{yy} & \xi_{yz} \\ \xi_{zx} & \xi_{zy} & \xi_{zz} \end{pmatrix} \begin{vmatrix} i & j & k \\ x - x_0 & y - y_0 & z - z_0 \\ F_x & F_y & F_z \end{vmatrix} \tag{39}$$

For the unity of the form of the Eq.s, it is advisable to bring the moment vector $\overline{M}$ (37), given as a vector product $(\bar{R} \times \bar{F})$, into a matrix form:

$$\begin{vmatrix} i & j & k \\ x-x_0 & y-y_0 & z-z_0 \\ F_x & F_y & F_z \end{vmatrix} = \begin{pmatrix} 0 & -(z-z_0) & y-y_0 \\ z-z_0 & 0 & -(x-x_0) \\ -(y-y_0) & x-x_0 & 0 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \quad (40)$$

Then Eq. (39) for the rotation angles is reduced to the form:

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} \xi_{xx} & \xi_{xy} & \xi_{xz} \\ \xi_{yx} & \xi_{yy} & \xi_{yz} \\ \xi_{zx} & \xi_{zy} & \xi_{zz} \end{pmatrix} \begin{pmatrix} 0 & -(z-z_0) & y-y_0 \\ z-z_0 & 0 & -(x-x_0) \\ -(y-y_0) & x-x_0 & 0 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \quad (41)$$

In folded form, it can be written as:

$$\overline{\omega} = \xi a_0 \bar{F} \quad (42)$$

where a special representation is introduced for the vector $\bar{R} = \overline{OA}$ in the form of a matrix:

$$a_0 = \begin{pmatrix} 0 & -(z-z_0) & y-y_0 \\ z-z_0 & 0 & -(x-x_0) \\ -(y-y_0) & x-x_0 & 0 \end{pmatrix} \quad (43)$$

In accordance with Eq. (43), the angular displacements of the point *A*, due to the rotation of the direction vector $\bar{l}$, are determined as the vector product $\overline{\omega} \times \bar{R}$. If we use the representation of the vector $\bar{R}$ in the form of a matrix a_0 (43), then it is impossible to directly perform the vector multiplication $\overline{\omega} \times \bar{R}$ (multiplying a vector by a matrix). Therefore, we first determine the opposite vector $\bar{R} \times \overline{\omega}$ (here, the matrix a_0 is multiplied by the vector $\overline{\omega}$). Taking into account (43), we have $\bar{R} \times \overline{\omega} = a_0 \xi a_0 \bar{F}$. Since the relation $\overline{\omega} \times \bar{R} = -\bar{R} \times \overline{\omega}$ is valid, expression (33) for the angular displacements of the point *A* will take the form:

$$\bar{\rho} = -a_0 \xi a_0 \bar{F} \quad (44)$$

For the total displacement of point *A* (plane-parallel displacement *r* of the base point *O* and displacement ρ due to rotation around the point *O*), in accordance with (3.227), we obtain:

$$\bar{u} = (e - a_0 \xi a_0) \bar{F} \quad (45)$$

This Eq. describes the elastic displacement of the body from the action of the force $\bar{F}$ taking into account the whole complex of factors characterizing the compliance of the bonds that fix the position of the body in space. We can call this movement full-factorial. The matrix *e* in this Eq. characterizes the compliance of the bonds that limit the plane-parallel movement of the body. Then the product of three matrices $-a_0 \xi a_0$ can be interpreted as the effective angular compliance matrix for point *A*. It characterizes the flexibility of the bonds that limit the angular displacement of point *A* relative to point *O*.

Conclusion. Matrix models of machining error in single-tool setups with a spatial arrangement of the tool are developed, taking into account the simultaneous action of all components of the cutting forces of the setup tool and elastic deformations of the technological system in all coordinate directions. These models are developed both in the distortion models of performed dimensions and in the scattering field models. A full-factor model of dimension distortion for single-carriage adjustment (setup) has been developed, which allows taking into account not only plane-parallel movements of technological subsystems, but also their angular movements around base points. Therefore, we believe that Eq. (45) can be used as the basis for a full-factor model of machining error. To do this, we first transform the analytical models of the elastic contact interaction of systems of bodies (18) to the level of full factorial ones. Analytical models (7), (14,15), (18) and (19) describe only plane-parallel displacements of contacting bodies. To take into account the entire range of movements in them, i.e. and angular displacements, it is sufficient to replace the plane-parallel displacement vectors of each contacting body $\overline{r_i}$ with the total displacement vectors $\overline{u_i}$. By applying the developed models to specific machining schemes, the issues of improving machining quality and productivity can be investigated through the management of technological parameters in adjustments used in modern CNC machines with wide technological capabilities. Also, within the technological capabilities of CNC machines, the issues of researching processing quality and durability of the cutting tool with various combinations of technological transitions, developing control matrix models of cutting conditions by fulfilling the requirement for the accuracy of dimensions in adjustments can be investigated.

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