MACHINE SCIENCE

ISSN: 2227-6912
E-ISSN: 2790-0479

2022, Volume 11, № 1. Pages 59 – 63

DYNAMICS OF THE CAM MECHANISM AT DELAYS AND LIMITED POWER-SUPPLY

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Abstract: The dynamics of a cam mechanism with delays in elasticity and friction is considered, the operation of which is supported by an energy source of limited power. The interaction between the cam mechanism and the energy source is described by nonlinear equations. To solve these equations, the method of direct linearization is used and the equations of non-stationary and stationary motions are derived. Relationships are obtained for calculating the stationary values of the amplitude and phase of oscillations, the speed of the energy source. A number of calculations have been performed in order to obtain information on the effect of delays on the dynamics of the cam mechanism.

Keywords: *cam, dynamics, oscillations, delay, elasticity, friction, energy source, limited excitation*.

Introduction. The determination of the parameters of the cam mechanism, taking into account its dynamics, is of great importance in the design. The functioning of the cam mechanism is supported by an energy source (engine), as a result of which their dynamics are also interconnected. Undesirable oscillatory processes that may occur during the operation of the cam mechanism also depend on the properties of the energy source that supports its operation. In this context, the well-known direction of the theory of oscillations comes to the fore, in which the interaction of an oscillatory system and an energy source is considered [1-2, etc.].

As is known [3-13 etc.], the characteristic of internal friction in materials, the imperfection of their elastic properties, etc. lead to a delay (hysteresis). It takes place in a number of devices (mills, vibrating machines, automatic control systems, conveyors, belt feeders, ball mills, flotation machines, drying drums, etc.) and technological processes. The delay leads to a deterioration in the dynamics and stability (up to loss) of the system. In this paper, the influence of delays in elasticity and friction on the dynamics of the cam mechanism is considered. It consists of an introduction, equations and their solutions, calculation results, conclusion.

Model and equations. In [14], the dynamics of the cam mechanism is considered on the basis of the model shown in Fig.1, where a round disk with an eccentricity ε acts as a cam. The disk is driven by an engine having a torque characteristic $M(\dot{\varphi})$, where $\dot{\varphi}$ is the speed of rotation. In the friction force arising in contact $F = f_1 N$ ($N = const$ is normal pressure force), the friction coefficient f_1 has a nonlinear characteristic

$$f_1(\dot{x}) = \sum_n b_n \dot{x}^n , \qquad b_n = const , \qquad n = 0, 1, 2, 3, 4, \dots \tag{1}$$

where $\dot{x}$ is speed of the pusher contact point.

Function (1) based on the direct linearization method [15, 16] is presented as

$$f_{1*}(\dot{x}) = B + k\dot{x} \tag{2}$$

$$B=\sum_n b_n S_n \upsilon^n, \qquad n=0,\,2,\,4,\dots \text{ (n is even number)}$$

$$k=\sum_n b_n \bar{S}_n \upsilon^{n-1}, \qquad n=1,\,3,\,5,\dots \text{ (n is odd number)}$$

where B and k are the linearization coefficients, $\upsilon=\max|\dot{x}|$, $S_n=(2r+1)/(2r+n+1)$, $\bar{S}_n=(2r+3)/(2r+n+2)$, r is *linearization accuracy parameter*, which is not limited, but it is enough to choose in the interval (0.2).

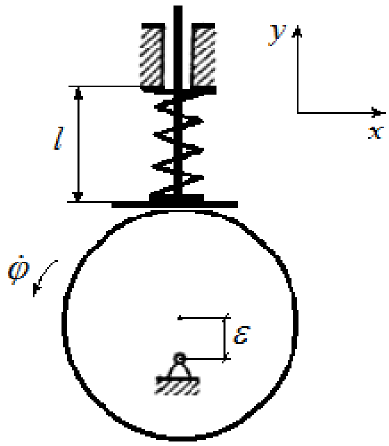

Figure1. System model

The motions of the system under the condition of continuous contact between the disk and the pusher are described [14] by the equations

$$m\ddot{x}+c(\varphi)x=F(\varphi,\dot{\varphi},\dot{x})$$
$$I\ddot{\varphi}=M(\dot{\varphi})-r(\varphi)F(\varphi,\dot{\varphi},\dot{x}) \tag{3}$$

where $F(\varphi,\dot{\varphi},\dot{x})=(B+k\dot{x})\left[N_0-y_0(c_y-m_T\dot{\varphi}^2)\cos\varphi+y_0k_н\dot{\varphi}\sin\varphi\right]$

$$c(\varphi)\approx c_0(1-\lambda\cos\varphi),\quad r(\varphi)\approx r_0-\mu+\varepsilon\cos\varphi+\mu\cos 2\varphi,\quad \mu=\varepsilon^2/4r_0,\quad N_0=B_y+c_y y_0.$$

In (3), the first equation describes the movement of the pusher, the second - the disk or energy source. Here m is the mass of the pusher brought to the point of contact, I is the moment of inertia of the disk, taking into account the mass of the rotating parts of the energy source, $r(\varphi)$ is the radius of the contact point of the disk and the pusher, B_y and c_y are the linearization coefficients of the nonlinear elastic force of the spring $f_2(y)=\sum_i c_i y^i$, $c_i=const$, $i=0,\,1,\,2,\,3$, c_0, λ, r_0, ε, y_0, $m_т$, $k_н$ are constant values.

In the presence of delays in friction $\dot{x}_\eta$ and elasticity x_τ, the first equation (3) takes the form

$$m\ddot{x}+k_\eta\dot{x}_\eta+c_\tau x_\tau+c_0(1-\lambda\cos\varphi)x=F(\varphi,\dot{\varphi},\dot{x}) \tag{4}$$

where $k_\eta=const$, $c_\tau=const$, $\dot{x}_\eta=\dot{x}(t-\eta)$, $x_\tau=x(t-\tau)$, $\eta=const$ and $\tau=const$ are delays.

Equation solutions. The action of the force $c(\varphi)x$ can lead to parametric oscillations. They are strongly manifested in the regions of parametric resonances, among which the most significant is the region of the main parametric resonance, where the ratio of the natural frequency to the frequency of the parametric excitation is about 1/2. Since the parametric excitation frequency is formed by the variable φ, which can be represented [17] as $\varphi = \Omega t + \hat{\varepsilon}_{oscil}$, where $\hat{\varepsilon}_{oscil}$ are the small oscillation components that are not taken into account, then in the region of the main parametric resonance $\Omega \approx 2\omega$, $\omega^2 = c_0/m$ is the natural frequency.

According to the method of change of variables with averaging [15], solutions (4), taking into account the delays $\dot{x}_\eta = -\upsilon \sin(\psi - p\eta)$, $x_\tau = a\cos(\psi - p\tau)$, have the form

$$x = a\cos\psi, \qquad \dot{x} = -\upsilon\sin\psi, \qquad \psi = pt + \xi \tag{5}$$

which, taking into account $p = \Omega/2$, give the following relations for determining the amplitude, phase of oscillations and speed of the energy source:

$$\frac{da}{dt} = \frac{a}{4m}(\delta\sin 2\xi + kG\cos 2\xi + L\Omega^{-1})$$

$$\frac{d\xi}{dt} = \frac{1}{4m}(\delta\cos 2\xi - kG\sin 2\xi + A\Omega^{-1}) \tag{6}$$

$$\frac{d\Omega}{dt} = \frac{1}{I}\left[M(\Omega) - B(R_0 N_0 - 0.5\varepsilon G)\right]$$

where $\upsilon = ap = a\Omega/2$, $\delta = ky_0 k_н \Omega - 2\lambda c_0 \Omega^{-1}$, $G = y_0(c_y - m_T\Omega^2)$, $R_0 = r_0 - \mu$,

$$L = 2\Omega(N_0 k - k_\eta \cos p\eta) + 4c_\tau \sin p\tau, \quad A = m(4\omega^2 - \Omega^2) + 2(k_\eta \Omega \sin p\eta + 2c_\tau \cos p\tau).$$

From (6) for $\dot{a} = 0$, $\dot{\xi} = 0$, $\dot{\Omega} = 0$ we obtain the equations for stationary motions

$$A^2 + L^2 = \Omega^2(k^2G^2 + \delta^2)$$

$$tg 2\xi = -(AkG - L\delta)/(LkG + A\delta) \tag{7}$$

$$M(\Omega) - S(\Omega) = 0$$

where $S(\Omega) = B(R_0 N_0 - 0.5\varepsilon G)$.

The $S(\Omega)$ expression determines the load on the energy source and the intersection points of the $M(\Omega)$ and $S(\Omega)$ curves give the stationary values of the speed Ω.

Calculations. Calculations were carried out to obtain information about the effect of lag on the dynamics of the cam system. The main design parameters are as follows: λ=0.02, $N_0 = 0.5$, $\omega = 1\,\text{c}^{-1}$, $c_0 = 1$ kgf $\cdot$ cm^{-1}, $c_\tau = 0.05$ kgf $\cdot$ cm^{-1}, $k_\eta = 0.02$ kgf $\cdot$ c $\cdot$ cm^{-1}. The linearization coefficients used $S_2 = 3/5$, $\bar{S}_3 = 3/4$, and for delays $p\eta$ and $p\tau$ values from the interval (0,2π).

Since the frequency difference is small in the resonance region, the approximate $\Omega \approx 2\omega$ was also used.

For calculations, the friction coefficient is chosen in the form

$$f_1(\dot{x}) = 0.303 + 0.0624\dot{x} + 0.648\dot{x}^2 - 0.18\dot{x}^3$$

which is a special case of the characteristic that is widespread [1-2,18-19, etc.] in practice $T(U) = q(1 - \alpha_1 U + \alpha_3 U^3)$ where $U = V - \dot{x}$, q, V, α_1, α_3 are constants and $V = 1.2\,\mathrm{cm}\cdot\mathrm{c}^{-1}$.

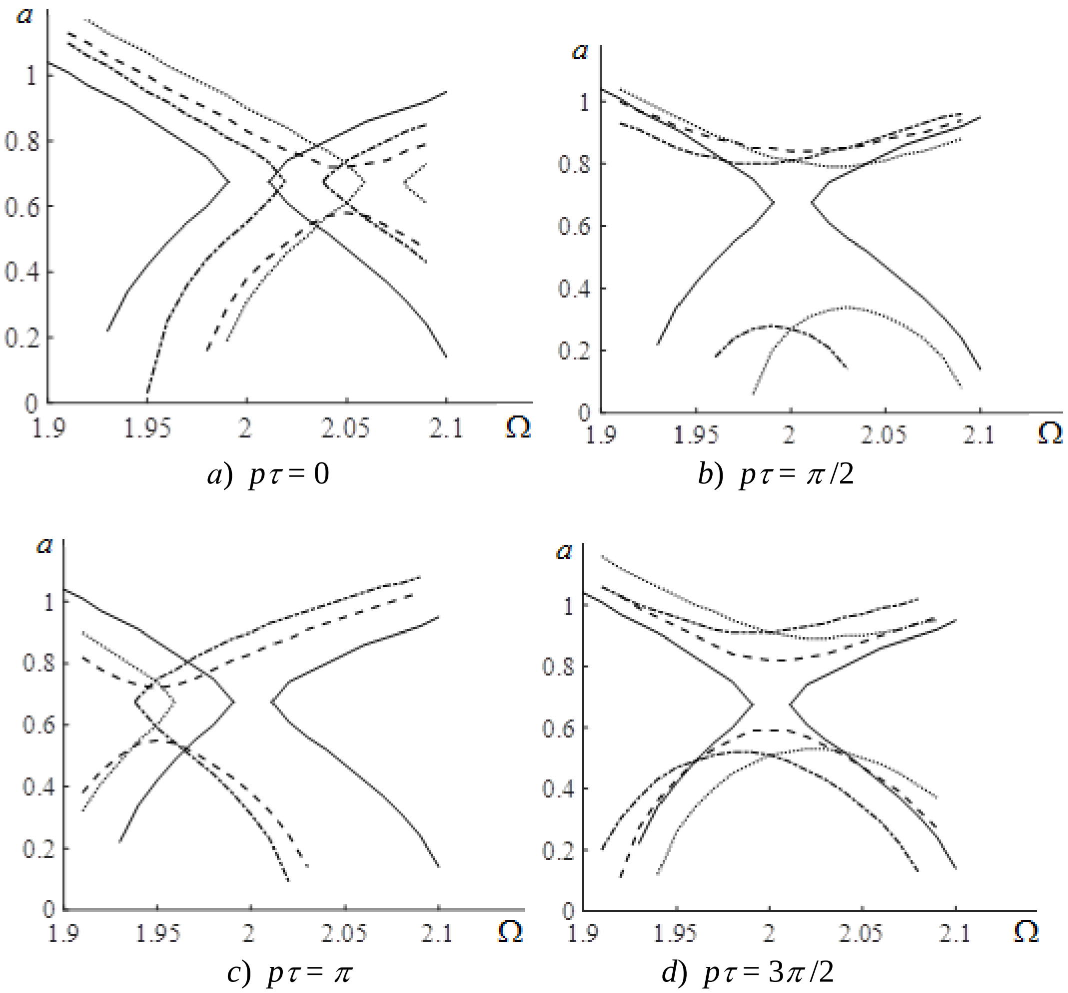

Figure 2. Amplitude-frequency curves:
solid curves – $c_\tau = 0$, $k_\eta = 0$, *double dotted –* $p\eta = \pi/2$,
dashed – $p\eta = \pi$, *dash-dotted –* $p\eta = 3\pi/2$.

Fig.2 shows the amplitude-frequency curves $a(\Omega)$ for different delays. Solid curves correspond to the absence of $c_\tau = 0$ and $k_\eta = 0$ delays, double dotted curves, $p\eta = \pi/2$, dashed curves, $p\eta = \pi$ and dash-dotted curves, $p\eta = 3\pi/2$. As can be seen from the figures, delays have a qualitative and quantitative effect on the amplitude-frequency curves, shift them in the frequency domain.

Conclusion. As follows from the above results, the combined action of various combinations of elasticity delay and damping can strongly influence the dynamics of the cam mechanism. Depending on various combinations of delay values, the oscillation amplitude undergoes qualitative and quantitative changes, the resonance zone can shift in frequency.

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Received: 01.06.2022

Accepted: 25.06.2022