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DETERMINATION OF THE DYNAMIC STATE OF SHIFT-DEFORMABLE SANDWICH NANOCOMPOSITE CYLINDRICAL PANELS

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Abstract: In this study, the dynamic behavior of shear deformable heterogeneous nanocomposite sandwich cylindrical panels containing carbon nanotube (CNT) patterned layers is investigated. The basic differential equations of the sandwich cylindrical panel composed of CNT patterned layers based on the Donnell type shell theory are derived. Then basic equations are solved by applying Galerkin method and obtained expression for the nondimensional free vibration frequency of three-layer nanocomposite cylindrical panels within the first order shear deformation shell theory (FOSDST). Finally, the influences of transverse shear strains, volume fractions, arrangement of sandwich nanocomposite layers on the nondimensional free vibration frequency are studied.

Keywords: *Carbon nanotube, nanocomposite, sandwich panel, free vibration, frequency, shear deformation theory*

Introduction. The typical sandwich structure consists of two surface layers containing a core layer [1]. The sandwich panels are one of the most used elements in structural applications and are frequently used especially in space vehicles, machinery, ship and automotive industries. In most aerospace applications, panels are used as coating elements in rocket systems as well as a part of pressurized fuel tank. The sandwich panels are one of structural elements that lead to optimum conditions in dynamic behavior. Investigation of vibration behavior of sandwich panels of any geometry plays an important role in successful applications of such structural elements.

The carbon nanotubes have recently attracted increased interest of researchers due to their unique properties in terms of strength, thermal stability, and electrical conductivity. CNTs were discovered experimentally by Japanese materials scientist Iijima in 1991 during the production of fullerenes by evaporation of arc discharge [2]. It has been experimentally proven that CNTs have outstanding mechanical properties compared to continuous carbon fibers [3, 4]. CNT reinforced composite materials are generally used in structural components of complex structural systems. Therefore, the dynamic behavior of structural components composed of CNT reinforced composites is extremely important for the efficient design process. Nanocomposites reinforced with CNTs, especially polymer-based nanocomposites are one of the most interesting research areas among nanocomposites. Many studies have shown that carbon nanotubes can be an effective tool for modifying the strength properties of polymer composite materials [5-7].

The first studies on the static and dynamic behavior of monolayer heterogeneous nanocomposite cylindrical panels belong to Liew et al. [8,9]. After this study, Kiani [10] investigated the dynamic behavior of functionally graded (FG)-CNT reinforced composite cylindrical panel under a moving load. Wang et al. [11] presented a semi-analytical method for vibration analysis of FG sandwich double-curved panels and rotary shells. In the research paper, Setoodeh and co-authors [12] examined the vibrational behavior of FG-CNT reinforced surface sheets and double-curved smart sandwich shells with FG porous cores. Di Sciuva and Sorrenti [13] analyzed the free vibration and buckling of functionally graded carbon nanotube reinforced sandwich plates using the Extended Refined Zigzag Theory. Avey et al. [14] carried out nonlinear vibration analysis of multilayer shell type structural elements with double curvature consisting of CNT patterned layers in different theories. The literature study reveals that the vibration problem of multi-layered structural elements consisting of CNT reinforced layers has not been sufficiently studied yet. In this study, the free vibration of sandwich cylindrical panels composed of CNT patterned layers is investigated within FOSDT.

Formulation of problem. The sandwich cylindrical panel composed of CNT patterned layers with side lengths a, radius R, and total thickness h is shown in Fig. 1. It is assumed that the sandwich cylindrical panel is composed of CNT patterned lamina of equal thickness. The lamina are perfectly bonded to each other, they do not slip, and all layers remain elastic during deformation. The main axes of elasticity of each lamina are assumed to be parallel to the coordinate axes on the reference surface. The curvilinear coordinate system $Oxyz$ is located on the reference surface and left corner of the cylindrical panel; where x and y axes are on the reference surface $z=0$ and the z axis is in the normal direction to the reference surface and is directed inward.

The effective material properties of each layer of sandwich panels with CNT patterns, based on the expanded rule of the mixture are expressed as follows [15]:

$$\begin{gathered}
Y_{11}^{(k)} = \eta_1^{(k)} V_{cn}^{(k)} Y_{11cn}^{(k)} + V_m^{(k)} Y_m^{(k)},\ Y_{22}^{(k)} = \frac{\eta_2^{(k)} Y_m^{(k)} Y_{22cn}^{(k)}}{Y_{22cn}^{(k)} V_m^{(k)} + Y_m^{(k)} V_{cn}^{(k)}},\ S_{12}^{(k)} = \frac{\eta_3^{(k)} S_m^{(k)} S_{12cn}^{(k)}}{S_{12cn}^{(k)} V_m^{(k)} + S_m^{(k)} V_{cn}^{(k)}} \\
S_{12}^{(k)} = S_{13}^{(k)} = 1.2 S_{23}^{(k)},\ p_{12}^{(k)} = V_{cn}^{*(k)} p_{12cn}^{(k)} + V_m^{(k)} p_m^{(k)},\ D_t^{(k)} = V_{cn}^{(k)} D_{cn}^{(k)} + V_m^{(k)} D_m^{(k)},\ (k=1,2,3)
\end{gathered} \tag{1}$$

where $Y_m^{(k)}, S_m^{(k)}, D_m^{(k)}, p_m^{(k)}$ are the elasticity modulus, density and Poisson's ratio in the layers of sandwich panels, and $Y_{ijcn}^{(k)},\ S_{ijcn}^{(k)},\ D_{cn}^{(k)},\ p_{12cn}^{(k)} (i,j=1,2,3)$ are the corresponding mechanical properties for the pattering CNT phase, respectively, $\eta_i^{(k)} (i=1,2,3)$ are the efficiency parameters in the layers of sandwich panels. Here $V_{cn}^{(k)}$ and $V_m^{(k)}$ are the volume fraction of CNTs in the layers of sandwich panels that obey the rule of $V_{cn}^{(k)} + V_m^{(k)} = 1$.

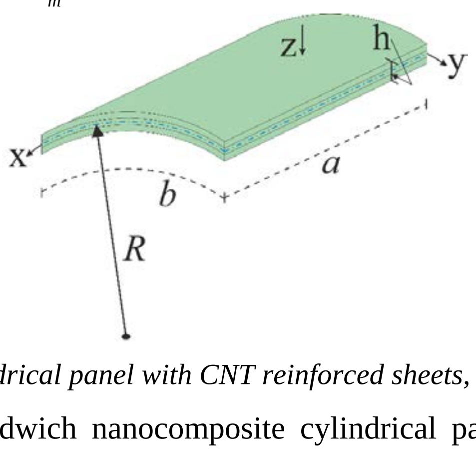
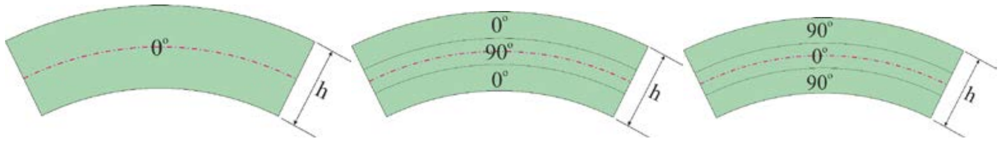


Fig. 1. Sandwich cylindrical panel with CNT reinforced sheets, and coordinate system

The cross-section of sandwich nanocomposite cylindrical panel is presented in Fig. 2, in which **(a)** 0-monolayer panel, **(b)** (0/90/0)-array sandwich panel, and **(c)** (90/0/90)-array sandwich panel.

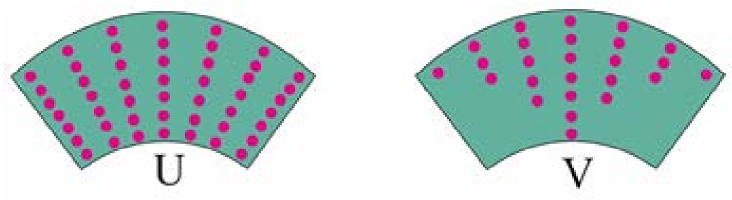


a) *b)* *c)*

Fig 2. The cross-section of sandwich nanocomposite cylindrical panel

The pattern of the volume fraction for CNTs over the thickness of the layers of sandwich panels as uniform pattern (U-pattern) and V-pattern is shown in Fig. 3.

$$V_{cn}^{(k)} = \begin{cases} \text{U} \quad at \;\; V_{cn}^{*(k)} \\ \text{V} \quad at \;\; 2\left(0.5-\overline{z}\right)V_{cn}^{*(k)}, \;\; \overline{z} = z/h \end{cases} \tag{2}$$



a) *b)*

Fig. 3. The cross-section of CNT patterned sheets (a) U-patterned, (b) B-patterned

The basic relations for the layers of sandwich panels patterned by CNTs within FOSDST can be defined as [14]:

$$\begin{bmatrix} \tau_{11}^{(k)} \\ \tau_{22}^{(k)} \\ \tau_{12}^{(k)} \\ \tau_{13}^{(k)} \\ \tau_{23}^{(k)} \end{bmatrix} = \begin{bmatrix} q_{11\overline{z}}^{(k)} & q_{12\overline{z}}^{(k)} & 0 & 0 & 0 \\ q_{21\overline{z}}^{(k)} & q_{22\overline{z}}^{(k)} & 0 & 0 & 0 \\ 0 & 0 & q_{66\overline{z}}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & q_{55\overline{z}}^{(k)} & 0 \\ 0 & 0 & 0 & 0 & q_{44\overline{z}}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix} (k=1,2,3) \tag{3}$$

where $\tau_{ij}^{(k)} (i=1,2;\, j=1,2,3, k=1,2,3)$ are the stresses in the layers of sandwich panels, $\varepsilon_{ij}(i,\, j=1,2,3)$ are the strains and $q_{ij\overline{x}_3}^{(k)},\, (i,\, j=1,2,6),$ denote material properties of CNT originating in the layers of sandwich panels.

The force and moments of sandwich panels composed of CNT originating layers are obtained as [16]:

$$\left(N_{ij},\, S_j\right) = \int_{-h/2}^{-h/6} \left(\tau_{ij}^{(1)}, \tau_{1j_1}^{(1)}\right) \mathrm{dz} + \int_{-h/6}^{h/6} \left(\tau_{ij}^{(2)}, \tau_{1j_1}^{(2)}\right) \mathrm{dz} + \int_{h/6}^{h/2} \left(\tau_{ij}^{(3)}, \tau_{1j_1}^{(3)}\right) \mathrm{dz}, \tag{4}$$

$$M_{ij} = \int_{-h/2}^{-h/6} \tau_{ij}^{(1)} \mathrm{zdz} + \int_{-h/6}^{h/6} \tau_{ij}^{(2)} \mathrm{zdz} + \int_{h/6}^{h/2} \tau_{ij}^{(3)} \mathrm{zdz},\; (i,\, j=1,2,\, j_1=2,3) \tag{5}$$

With the Airy stress function Φ, the in-plane forces are defined as [16]:

$$T_{11} = h\frac{\partial^2 \Phi}{\partial y^2}, T_{22} = h\frac{\partial^2 \Phi}{\partial x^2}, T_{12} = -h\frac{\partial^2 \Phi}{\partial x \partial y} \tag{6}$$

Substituting (4) into (5) and then substituting the resulting expressions and the relation (6) in the basic equations [14], the governing equations of sandwich panels with CNT origin can be obtained as:

$$\begin{aligned} &L_{11}(\Phi)+L_{12}(w)+L_{13}(\psi_1)+L_{14}(\psi_2)=0\\ &L_{21}(\Phi)+L_{22}(w)+L_{23}(\psi_1)+L_{24}(\psi_2)=0\\ &L_{31}(\Phi)+L_{32}(w)+L_{33}(\psi_1)+L_{34}(\psi_2)=0\\ &L_{41}(\Phi)+L_{42}(w)+L_{43}(\psi_1)+L_{44}(\psi_2)=0 \end{aligned} \tag{7}$$

where $L_{ij}(i=1,2,...,4,\ j=1,2,..,4)$ are differential operators and defined in ref. [14].

The following approximation functions is sought for sandwich panels containing CNT reinforced layers:

$$\begin{aligned} &w = f(t)\sin(m_1x)\sin(n_1y),\ \Phi = f_0(t)\sin(m_1x)\sin(n_1y),\\ &\psi_1 = f_1(t)\cos(m_1x)\sin(n_1y),\ \psi_2 = f_2(t)\sin(m_1x)\cos(n_1y) \end{aligned} \tag{8}$$

where $f(t)$ and $f_i(t)$ $(i=0,1,2)$ are the functions of a time, $m_1=\frac{m\pi}{a}$, $n_1=\frac{n\pi}{b}$, in which (m,n) are the wave numbers.

After substituting the functions of (8) into the system of Eqs. (7), the Galerkin method is applied and the unknowns $f_i(t)$ $(i=0,1,2)$ are eliminated from the resulting system of equations, the following expression is obtained for the free vibration frequency sandwich panels containing CNT reinforced layers in the framework of FOSDST:

$$\Omega_{sdt}=\sqrt{\frac{u_4u_1-u_2u_3}{\bar{D}_tu_1}} \tag{9}$$

where $\bar{D}_t=\int_{-h/2}^{-h/6}D_t^{(1)}\mathrm{d}z+\int_{-h/6}^{h/6}D_t^{(2)}\mathrm{d}z+\int_{h/6}^{h/2}D_t^{(3)}\mathrm{d}z$, $u_i(i=1,2,..,4)$ are parameters depending on the properties of the sandwich panels containing the CNT reinforced layers, and the following expression is used for the nondimensional frequency parameter:

$$\Omega_{1sdt}=\Omega_{sdt}h\sqrt{\frac{D_m^{(1)}}{E_m^{(1)}}} \tag{10}$$

The expressions (9) and (10) transform into expressions of dimensional and nondimensional frequencies within the framework of classical shell theory (CST), when the transverse shear stresses $\tau_{13}^{(k)}$ and $\tau_{23}^{(k)}$ in the layers of the sandwich panel are not taken into account in the basic relations.

Analysis of the obtained results. In the numerical analysis, A poly (methyl methacrylate) called PMMA reinforced with (10,10) single-walled CNTs is used. The elastic properties of the PMMA matrix are as follows: $Y_m^{(k)}=2.5\,\mathrm{GPa}$, $p_m^{(k)}=0.34$ and $D_m^{(k)}=1150\,\mathrm{kg/m^3}$ (k=1 and 3). The geometry and elastic properties of CNT are defined as: $r=9.26\,\mathrm{nm}$, $a_1=0.68\,\mathrm{nm}$, $h_1=0.067\,\mathrm{nm}$ and $Y_{11}^{cn(k)}=5.6466\,\mathrm{TPa}$, $Y_{22}^{cn(k)}=7.08\,\mathrm{TPa}$, $S_{12}^{cn(k)}=1.9445\,\mathrm{TPa}$, $p_{12}^{cn(k)}=0.175$, $D_{cn}^{(k)}=1400\,\mathrm{kg/m^3}$. The total volume fractions and productivity parameters of CNTs in the layers are defined as follows: $\eta_1^{(k)}=0.137$, $\eta_2^{(k)}=1.022$, $\eta_3^{(k)}=0.715$ at $V_{cn}^{*(k)}=0.12$, $\eta_1^{(k)}=0.142$, $\eta_2^{(k)}=1.626$, $\eta_3^{(k)}=1.138$ at $V_{cn}^{*(k)}=0.17$ and $\eta_1^{(k)}=0.141$, $\eta_2^{(k)}=1.585$, $\eta_3^{(k)}=1.109$ at $V_{cn}^{*(k)}=0.28$

[15]. The shear stresses of sandwich panels containing CNT patterned layers are used as, $\varphi_j^{(k)}(z) = z(1-4z^2/3h^2)$, $(j=1,2)$ [16].

The variation of nondimensional free frequency parameters of nanocomposite sandwich panels for three types of sequences of layers with U-and V-patterns depending on the R/a ratio is tabulated in Table 1. The following data are used in the computations: $a/b=1$, $a/h=20$, $V_{cn}^{*(k)}=0.12$ and $(m,n)=(1,1)$. The sandwich panel consists of (0/90/0) and (90/0/90)-arranged layers, and calculations are also made for the (0)-monolayer panel for the comparison. As can be seen from Table 1, the nondimensional free vibration frequency values decrease due to the increase of R/a in all the heterogeneous nanocomposite panels containing CNT layers with (0), (0/90/0) and (90/0/90)-arrays. The free frequency values in sandwich panels containing U-patterned layer are higher than the frequency values for V-patterned sandwich panels. Depending on the increase in the R/a ratio, the shear deformation effect on the frequency values increases. The highest shear deformation effect (90/0/90)-arrayed U-patterned sandwich panel is 14.22 % for R/a =3, the least effect is (0)-arrayed occurs in the monolayer panel at R/a =1 (9.40%). Despite the decrease the effect of shear deformations on frequency values for V-patterned layers, it maintains its importance. For example, the highest shear deformations effect in (90/0/90)-arrayed U-patterned sandwich panel is 9.53% at R/a =3 and the least effect is at R/a =1 in (0)-array monolayer panel occurs (5.94%). Also, in (0/90/0)-patterned sandwich panels, the highest effect is 11.99% at R/a=3 in the U-pattern, while the least effect occurs at R/a =1 in V-pattern sandwich panels (7.41%).

Table 1. The variation of nondimensional free frequency parameters of sandwich panels for three types of sequences of layers with U-and V-patterns depending on the R/a

		U-pattern		V-pattern	
Sequences	R/a	$10\Omega_{1sdt}$			
0/0/0	1.0	0.464	0.512	0.430	0.457
	1.5	0.428	0.480	0.388	0.418
	2.0	0.414	0.468	0.372	0.403
	2.5	0.408	0.462	0.364	0.396
	3	0.404	0.459	0.360	0.391
0/90/0	1.0	0.465	0.514	0.433	0.462
	1.5	0.429	0.481	0.392	0.424
	2.0	0.415	0.469	0.376	0.409
	2.5	0.409	0.464	0.369	0.402
	3	0.405	0.461	0.364	0.398
90/0/90	1.0	0.490	0.554	0.449	0.484
	1.5	0.455	0.524	0.409	0.447
	2.0	0.443	0.513	0.394	0.433
	2.5	0.437	0.508	0.387	0.427
	3	0.434	0.505	0.382	0.423

In the (0)-monolayer panel, when the R/a ratio increases, the V-pattern effect on the frequency values increases from (-7.38%) to (-11.01%) within SDT, while this effect increases from (-10.79%) to (-14.67%) under the CST. When the R/a ratio increases in (0/90/0) layered sandwich panel, the V-pattern effect on the frequency values increases from (-6.81%) to (-10.11%) in the framework of the SDT, while this effect increases from (-10.07%) to (-13.63%) within the CST. In the (90/0/90)-layered sandwich panel, when the R/a increases, the V-pattern effect on the frequency values increases from (-8.29%) to (-11.78%) in the framework of SDT, while this effect increases from (-12.64%) to (16.35%) within CST.

Conclusion. In this study, the free vibration of sandwich cylindrical panels composed of CNT patterned layers is investigated within FOSDST. The governing equations of sandwich cylindrical panels composed of CNT patterned layers based on the Donnell type shell theory are derived. Then solved by applying the Galerkin method and obtained expression for the frequency based on the FOSDST. The effects of transverse shear strains, volume fraction, sequence of nanocomposite layers on the frequency are discussed. The analyzes and interpretations carried out revealed that the effects of above-mentioned factors on the nondimensional vibration frequency are very important and it is necessary to consider these factors during the design of sandwich nanocomposite panels.

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