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FREE VIBRATION ANALYSIS OF THE SANDWICH COMPLETE CONICAL SHELL WITH FUNCTIONALLY GRADED MATERIALS FACINGS

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Abstract: The free vibration analysis of the sandwich complete conical shell with functionally graded materials (FGMs) facings based on the classical shell theory (CST) is studied. The basic equations of metal-rich truncated conical shells with FGMs facings are derived and solved by employing Galerkin method. The closed form solution for dimensionless frequency parameters of the sandwich complete conical shell with FGM facings is found. The effects of kinds of FGM facings and variation of thickness of facings on the magnitudes of dimensionless frequency parameters are studied in numerical analysis part.

Keywords: *Functionally graded materials, FGM facings, sandwich complete conical shell, metal-rich core, free vibration, frequency parameters*

Introduction. Aircraft, space vehicles, satellites and sensitive technologies, such as nuclear reactors, innovative high-performance carrier component design have always been at the forefront. Efforts to develop a new generation of composite materials in the form of coating or basic material, makes it possible to create new sandwich structures and makes it more popular. Such structures, good thermal and acoustic insulation, to reduce some resistance, light weight and long service life, high strength and hardness, both features, and also be easy to produce [1, 2].

One of the main drawbacks of conventional sandwich constructions is that delamination occurs due to strong stress concentration on the surfaces between the layers. For this reason, the designer needs to systematically expand the class of materials in order to select the best materials for the sandwich construction elements. In recent years, functional graded materials (FGMs) produced due to its wide range of applications belong to the new generation of composite materials [3-6]. Smooth and continuous change of FGM properties from one surface to another reduces stress concentration in layered composites and contributes to avoid interface problems. One of first studies on the design, vibration and stability of sandwich plates and shells with FDM layer is presented in the source [7, 8]. Following these studies, stability and vibration problems of sandwich construction elements are solved using different theories [9-14]. In this study, the closed-form solution is found for the dimensionless frequency of the FGM coated sandwich complete conical shells and the aforementioned drawbacks can be slightly overcome in the literature.

Basic relations. Figure 1a shows a sandwich truncated conical shell with a metal core and FGM coatings. This sandwich-truncated conical shell is transformed into a complete sandwich-conical shell (Figure 1b). A sandwich conical shell with

FGM coatings and enriched metal cores are formed in a triple system. The $OS\zeta\theta$ coordinate system is considered at the top of the complete cone and on the reference surface of the three-layer system. The axes and some sandwich conical shell parameters are explained on the figure (Fig. 1a). Length of complete and truncated conical shells are S_2 and L, the radii of small large bases are R_1 and R_2, respectively. The total thickness of sandwich conical shells are $h = 2h_{fg} + h_m$ in which h_m is thickness of metal core and h_{fg} is the thickness of FGM coatings (Fig. 1c).

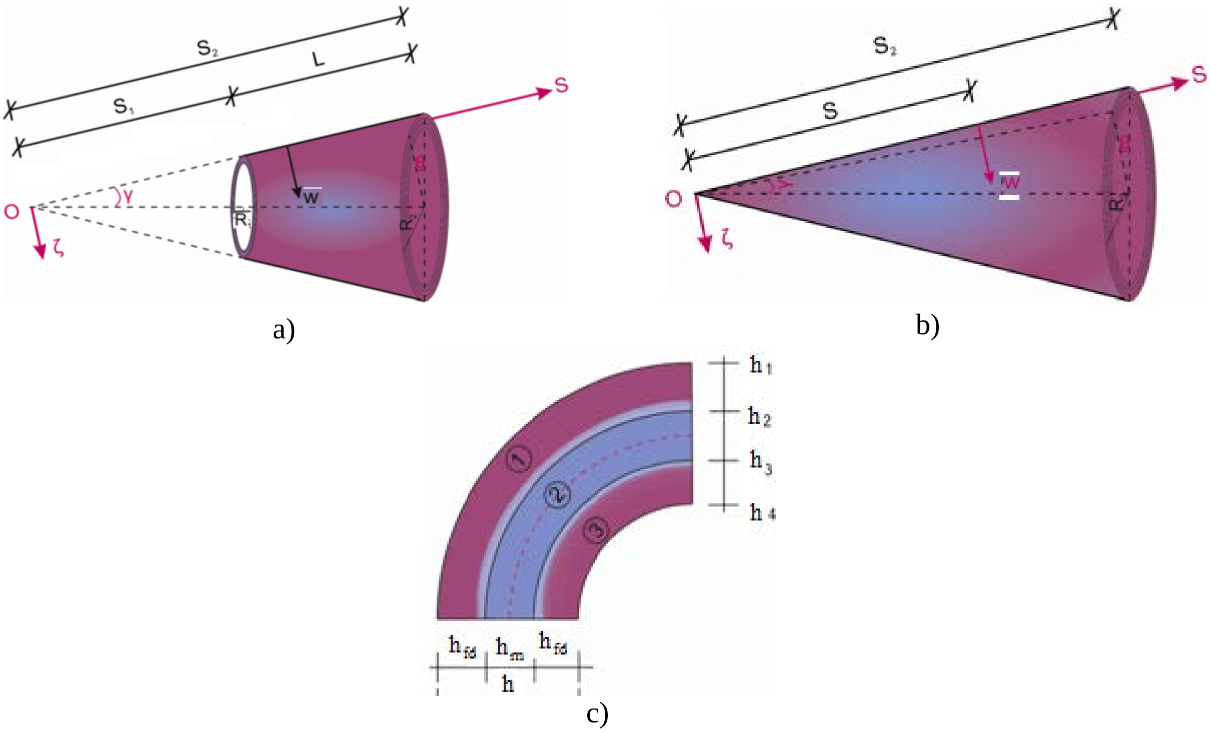

Figure 1. a) Coordinate system in the FGM sandwich truncated and b) complete conical shells, c) arrangement of layers; (1) FGM, (2) enriched metal, (3) FCM

The volume fractions of the materials in the layers of FGM coated sandwich conical shells are expressed as [11]:

$$V^{(1)} = (2\bar{\zeta} + 1)/(2\zeta_2 + 1), \quad 1.layer$$

$$V^{(2)} = 1 \qquad 1, \qquad\qquad 2.layer \tag{1}$$

$$V^{(3)} = (2\bar{\zeta} - 1)/(2\zeta_3 - 1), \quad 3.\ layer$$

The mathematical models of the material properties of FGM coated sandwich conical shells are expressed as [11]:

$$E_{fd}^{(1)} = E_c e^{V^{(1)}\ln(E_m/E_{cs})}, \quad \nu_{fd}^{(1)} = \nu_c e^{V^{(1)}\ln(\nu_m/\nu_c)}, \ \rho_{fd}^{(1)} = \rho_c e^{V^{(1)}\ln(\rho_m/\rho_c)}$$

$$E_{fd}^{(3)} = E_c e^{V^{(3)}\ln(E_m/E_c)}, \quad \nu_{fd}^{(3)} = \nu_c e^{V^{(3)}\ln(\nu_m/\nu_c)}, \ \rho_{fd}^{(3)} = \rho_c e^{V^{(3)}\ln(\rho_m/\rho_c)} \tag{2}$$

where $E_{\mathrm{m}}, \nu_m, \rho_m$ and $E_{\mathrm{c}}, \nu_c, \rho_c$ are the Young's moduli, Poisson's ratios and densities of the metal and ceramic surfaces of the FGM coatings in the sandwich conical shells, respectively.

$$[E_{sand}, \nu_{sand}] = \begin{cases} E_{fd}^{(1)}, \nu_{fd}^{(1)}, \ \rho_{fd}^{(1)} & -h_1 \le \zeta \le -h_2 \\ E_m, \nu_m, \ \rho_m & -h_2 < \zeta < h_3 \\ E_{fd}^{(3)}, \nu_{fd}^{(3)}, \ \rho_{fd}^{(3)} & h_3 \le \zeta \le h_4 \end{cases} \tag{3}$$

Basic Equations. The force and moment components are defined as [7-12]:

$$\left(T_{ij}, M_{ij}\right) = \sum_{k=1}^{3} \int_{h_k}^{h_{k+1}} [1, \zeta] \sigma_{ij}^{(k)} \, \mathrm{d}\zeta \quad (i, j = 1,2) \tag{4}$$

where T_{ij} and $M_{\mathrm{ij}} (i = 1,2)$ are force and moment components, $\sigma_{ij}^{(k)} (i = 1,2; j = 2,3)$ are stresses on the layers of the sandwich conical shells with FGM coatings and $k = 1,2,3$ denotes number of layers.

The relations between the stress-deformation components of the FGM-coated sandwich conical shells can be written as:

$$\begin{bmatrix} \sigma_{11}^{(k)} \\ \sigma_{22}^{(k)} \\ \sigma_{12}^{(k)} \end{bmatrix} = \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} & 0 \\ Q_{12}^{(k)} & Q_{11}^{(k)} & 0 \\ 0 & 0 & Q_{66}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} \tag{5}$$

where $Q_{ij}^{(k)}, (i, j = 1,2,...,6; k = 1,2,3)$, are the coefficients in the layers that contain the quantities depending on the material properties [11-14].

The relations between the force components and the Airy function are as follows [6, 12-14]:

$$T_{11} = h \frac{\partial^2 \Psi}{\partial y^2}, T_{22} = h \frac{\partial^2 \Psi}{\partial x^2}, T_{12} = -h \frac{\partial^2 \Psi}{\partial x \partial y} \tag{6}$$

Substituting the relations (5) into integrals (4), then resulting equations with the relations (6), in motion and compatibility equations, after some mathematical operations we obtain the motion and compatibility equations for FGM sandwich truncated conical shells as

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \Psi \\ w \end{bmatrix} = 0 \tag{7}$$

The contents of the above differential operators are explained as:

$$\left.\begin{aligned}
L_{11} &= C_{02}e^{-4x}\left(\frac{\partial^4}{\partial x^4}-4\frac{\partial^3}{\partial x^3}+4\frac{\partial^2}{\partial x^2}+\frac{\partial^4}{\partial \varphi^4}+2\frac{\partial^2}{\partial \varphi^2}\right) \\
&\quad +(2C_{01}-C_{05})e^{-4x}\left(\frac{\partial^4}{\partial x^2\partial \varphi^2}-2\frac{\partial^3}{\partial x\partial \varphi^2}+\frac{\partial^2}{\partial \varphi^2}\right)+S_2e^{-3x}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial}{\partial x}\right)\cot\,\gamma \\
L_{12} &= -C_{03}e^{-4x}\left(\frac{\partial^4}{\partial \varphi^4}+2\frac{\partial^2}{\partial \varphi^2}+\frac{\partial^4}{\partial x^4}-4\frac{\partial^3}{\partial x^3}+4\frac{\partial^2}{\partial x^2}\right) \\
&\quad -(2C_{04}+C_{06})e^{-4x}\left(\frac{\partial^4}{\partial x^2\partial \varphi^2}+2\frac{\partial^3}{\partial x\partial \varphi^2}-\frac{\partial^2}{\partial \varphi^2}\right)-\rho_t S_2^4\frac{\partial^2}{\partial t^2} \\
L_{21} &= B_{01}e^{-4x}\left(\frac{\partial^4}{\partial x^4}-4\frac{\partial^3}{\partial x^3}+4\frac{\partial^2}{\partial x^2}+\frac{\partial^4}{\partial \theta_1^4}+2\frac{\partial^2}{\partial \varphi^2}\right) \\
&\quad +(2B_{02}+B_{05})e^{-4x}\left(\frac{\partial^4}{\partial x^2\partial \varphi^2}-2\frac{\partial^3}{\partial x\partial \varphi^2}+\frac{\partial^2}{\partial \varphi^2}\right) \\
L_{21} &= -B_{04}e^{-4x}\left(\frac{\partial^4}{\partial \varphi^4}+2\frac{\partial^2}{\partial \varphi^2}+\frac{\partial^4}{\partial x^4}-4\frac{\partial^3}{\partial x^3}+4\frac{\partial^2}{\partial x^2}\right) \\
&\quad +(B_{06}-2B_{03})e^{-4x}\left(\frac{\partial^4}{\partial x^2\partial \varphi^2}-2\frac{\partial^3}{\partial x\partial \varphi^2}+\frac{\partial^2}{\partial \varphi^2}\right)+S_2e^{-3x}\cot\,\gamma\left(\frac{\partial^2}{\partial x^2}-\frac{\partial}{\partial x}\right)
\end{aligned}\right\} \tag{8}$$

where $\phi=\theta\sin\gamma$ and $x=\ln(S/S_2)$ the new variable that used to facilitate the integration process and $C_{0i},B_{0i}(i=1,2,...,6)$, are parameters depending on the material properties of FGM sandwich conical shells [11].

Solution of Basic Equations. Since the FGM sandwich conical shell is the simply supported, the solution of the Eq. (7) is searched as follows [11, 12]:

$$\Psi(x,\theta,t)=\overline{\Psi}(t)S_2\,e^{(\lambda+1)x}\sin(m_1\mathrm{x})\cos(m_2\phi),\; w(x,\theta,t)=\overline{w}(t)e^{\lambda x}\sin(m_1\mathrm{x})\cos(m_2\phi) \tag{9}$$

where $\overline{\Psi}(t)$ and $\overline{w}(t)$ time dependent functions and λ is a parameter which defined from minimum conditions of the frequency and it takes the value 2.4. In addition, $m_1=\frac{m\pi}{x_0},\ m_2=\frac{n}{\sin\gamma},\ x_0=\ln\frac{S_2}{S_1}$ in which m is the meridional wave number and n is the circumferential wave number.

Substituting (9) into set of the Eq. (7) and taken into account $S_1\to 0$, then applying Galerkin's method, after some mathematical operations we obtain the expression for he dimensionless frequency :

$$\omega_1=\frac{S_2}{h}\sqrt{\frac{\Lambda_1+\Lambda_2}{\Lambda_3\Lambda_\rho}\frac{\rho_c}{E_c}} \tag{10}$$

where the following definitions apply:

$$\left.\begin{aligned}
&\Lambda_1=\Big[C_{02}\left(A_1-4A_2+4A_3+m_2^4A_5-2m_2^2A_8\right)+(2C_{01}-C_{05})m_2^2\left(2A_4-A_3-A_5\right)+\left(A_{01}-A_{00}\right)S_2\cot\gamma\Big]\times\\
&\quad\times[B_{03}\left(4B_{14}-m_2^4B_9+2m_2^2B_{12}-B_{13}-4B_{15}\right)-(B_{06}-2B_{03})m_2^2\left(B_{10}+2B_{11}\right)+B_{00}S_2\cot\ \gamma];\\
&\Lambda_2=\Big[C_{03}m\left(\beta_2^4A_9-2m_2^2A_{12}+A_{13}-4A_{14}+4A_{15}\right)+(2C_{04}+C_{06})m_2^2\left(2A_{11}-A_{10}-A_9\right)\Big]\\
&\quad\times\Big[B_{01}\left(m_2^4B_1-2m_2^2B_4+B_5-4B_6+4B_7\right)-(2B_{02}+B_{05})m_2^2\left(B_2+2B_3-B_4\right)\Big];\\
&\Lambda_3=B_{01}\left(m_2^4B_1-2m_2^2B_4+B_5-4B_6+4B_7\right)-(2B_{02}+B_{05})m_2^2\left(B_2+2B_3-B_4\right);\\
&\Lambda_\rho=2\theta_{2\lambda+2}\rho_tS_2^4,\ \theta_{2\lambda+i}=\frac{m_1^2}{\left[(2\lambda+i)^2+4m_1^2\right](2\lambda+i)};\ i=-2;\,-1;\ 0;\ 1;\ 2
\end{aligned}\right\}\quad(11)$$

In order to find the minimum values of the dimensionless frequency parameter of the FGM sandwich complete conical shells, the expression (10) is minimized according to the number of wave, n.

Results and discussion. In this section, the minimum values of the dimensionless frequency parameters of FGM sandwich complete conical shells are found numerically and the results obtained are analyzed. As the FGM facings, a mixture of SUS304/Si_3N_4, i.e., stainless steel and silicon nitrate, which is a metal-ceramic mixture, is used. The material properties of FGM facings are taken from Ref. [6]. Sandwich complete conical shells with FGM, ceramic and metal coatings are used and three types of sandwich cones are designed as follows: FMF, CMC and MMM.

In Table 1, the variation of the values of ω_1 for three types of FMF, CMC and MMM sandwich complete conical shells versus half-peak angle, γ, is presented. In Table 1, the geometric dimensions of the sandwich complete conical shells are as follows: h_{core}/h_{coat}=4, $R_2/h=25$ and $\lambda=2.4$. Also, the values in parentheses indicate the number of circumferential wave number (n).

As can be seen in Table 1, the values of ω_1 decrease considerably in all three types of sandwich complete conical shells due to the increase of half-peak angle, γ, while the number of circumferential waves increases. When the values of the ω_1 for the FMF sandwich complete conical shell are compared with the values of CMC and MMM cones, the effects of FGM coatings are (-11%) and 29%, respectively, and approximately independent of the change of the half-peak angle, γ.

Table 1. Variation of the values of ω_1 for FMF, CMC and MMM complete conical shells versus the γ.

ω_1					
γ	15°	30°	45°	60°	75°
FMF	44.061(2)	23.618(3)	13.643(3)	9.494(3)	6.790(3)
CMC	49.409(2)	26.578(3)	15.299(3)	10.640(3)	7.634(3)
MMM	38.200(2)	20.324(3)	11.826(3)	8.242(3)	5.854(3)

Fig. 2 presents the distribution of the values of ω_1 for three types (FMF, CMC and MMM) sandwich complete conical shells depending on the ratio of the coating thickness (h_{core}/h_{coat}) to the core thickness. The geometric dimensions of the sandwich complete conical shells are as follows: $R_2/h = 25$, $\gamma = 30^o$ and $\lambda = 2.4$.

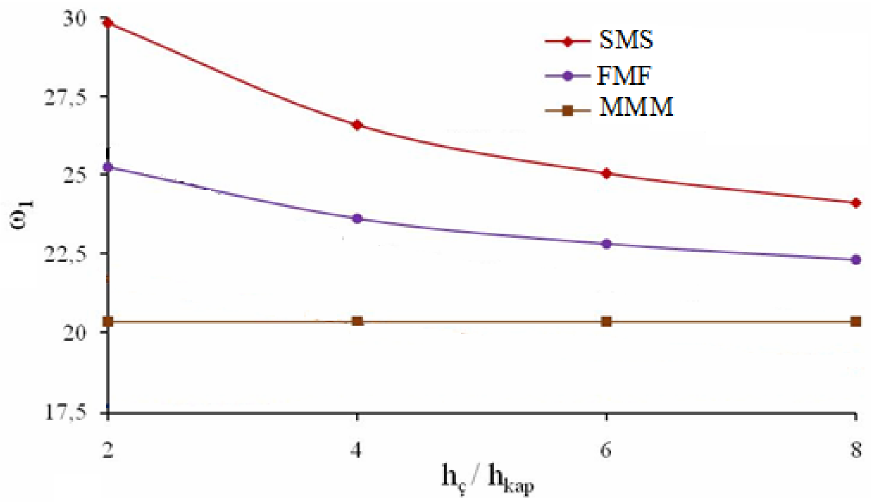

Figure 2. *Variations of the values of ω_1 for FMF, CMC and MMM sandwich complete conical shells versus the h_{core}/h_{coat}*

As shown in Figure 2, the values of ω_1 for FMF and CMC sandwich conical shells decrease when the ratio, h_{core}/h_{coat}, increases. When the values of ω_1 for FMF and CMC sandwich conical shells were compared, it was found that the effect of FMF coating profile on the values of ω_1 decreased significantly (- 15.4%) for $h_{core}/h_{coat} = 2$, even though it was approximately (-8%) for $h_{core}/h_{coat} = 8$. Thus, the effect of the FGM coatings on the values of ω_1 is reduced by the increase of the ratio, h_{core}/h_{coat}. When the values of the dimensionless frequency parameter of FMF sandwich conical shells were compared with the values of the MMM complete conical shell, the effect of FGM coatings was 23.61% for $h_{core}/h_{coat} = 2$, whereas this effect was reduced to 9%, when $h_{core}/h_{coat} = 8$. When the dimensionless frequency parameter values of CMC sandwich complete conical shells were compared to the values of the MMM complete conical shell, the effect of metal and ceramic coatings was 46% for $h_{core}/h_{coat} = 2$, while this effect was found to be 18.5%, when $h_{core}/h_{coat} = 8$.

Conclusions. The free vibration analysis of the sandwich complete conical shell with FGMs facings based on the CST is studied. The basic equations of metal-rich truncated conical shells with FGMs facings are derived on the basis of CST and solved by employing Galerkin method. The closed form solution for dimensionless frequency parameters of the sandwich complete conical shell with FGM facings is found. The effects of kinds of FGM facings and variation of thickness of facings on the magnitudes of dimensionless frequency parameters are studied in numerical analysis part.

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