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FUNCTION GENERATION OF A WATT II TYPE PLANAR MECHANISM WITH PRISMATIC OUTPUT USING DECOMPOSITION AND CORRECTION METHOD

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Abstract: The method of decomposition is a useful method for function generation with multi-loop mechanisms. The recently introduced correction methods applied together with the method of decomposition allows the designer to cancel out the errors in the first loop of a two-loop mechanism with the errors in the second loop. In this study, the decomposition and correction method is applied for a Watt II type planar six-link mechanism with prismatic output. Five design parameters are defined for each loop resulting in ten design parameters in total. The design parameters are determined analytically. The generation error is decreased by adjusting free parameters such as the limits of the some joint angles and parameters due to the decomposition of the function to be generated, while considering several constraints such as link lengths ratios and ranges of the joint variables. The success of the method is illustrated with a numerical example.

Keywords: *Function generation, decomposition and correction method, planar Watt II mechanism with prismatic output.*

Introduction.

Recently Kiper et al. [1] have introduced a new kinematic synthesis method for function generating multi-loop mechanisms based on decomposition and correction methods. For a two-loop mechanism, the decomposition method decomposes a function y = f(x) into two as w = g(x) and y = h(w) = h(g(x)) = f(x) [2]. The loops of a two-loop mechanism are to be used to generate the decomposed functions w = g(x) and y = h(w). In general for mechanisms with more than two successive loops, the function to be generated, y = f(x), may be decomposed into as many functions as the number of loops. The three different correction methods introduced in [1] aim neutralizing the generation error of the first loop by matching the errors due to the second loop. The correction method can be generalized for mechanisms with more than two loops as well. Kiper et al. compare their methods with other function generation methods in the literature [3, 4] and demonstrate the superiority of their methods for generation with less errors.

In [1], a Watt II type planar six-link mechanism with revolute joints only is used for an application of the decomposition and correction methods. Via numerical examples it is demonstrated that in general correction methods #2 and #3 provide superior results compared to correction method #1. Correction method #3 requires making use of the derivative of the loop closure equations and hence it is a relatively more complex method to apply. Therefore we choose to use correction method #2 in this study to formulize the function generation of a Watt II type planar six-link mechanism which comprises six revolute joints and a prismatic joint. The prismatic joint is the output of the mechanism. Such mechanisms are quite common in applications, where the first loop is a crank-rocker type four-bar loop and the second loop is a slider-crank loop. Some deep drawing, blanking and knuckle-joint presses comprise such mechanisms. It is not a straightforward task to formulize the design of such mechanisms as a function generation synthesis problem and such formulizations are kept out of scope of this paper.

The paper is organized as follows: The description of the Watt II type six-link planar mechanism with prismatic output and the formulation for function generation is presented in Section 2. The correction method is explained in Section 3. A numerical example is given in Sections 4. Section 5 concludes the paper.

The Mechanism and Function Generation Problem Definition. The Watt II type planar six-link mechanism in this study is comprises two ternary and four binary links connected to each other by six revolute joints and a prismatic joint (Fig. 1). The I/O equation of a four-bar mechanism is not affected by the scale of the mechanism, and the four-bar loop A_0ABB_0 can be scaled independent from the slider-crank loop B_0CD, so we assume $|A_0B_0| = 1$. Once the synthesis task is done, the designer can scale the four-bar loop A_0ABB_0 with any desired scale ratio.

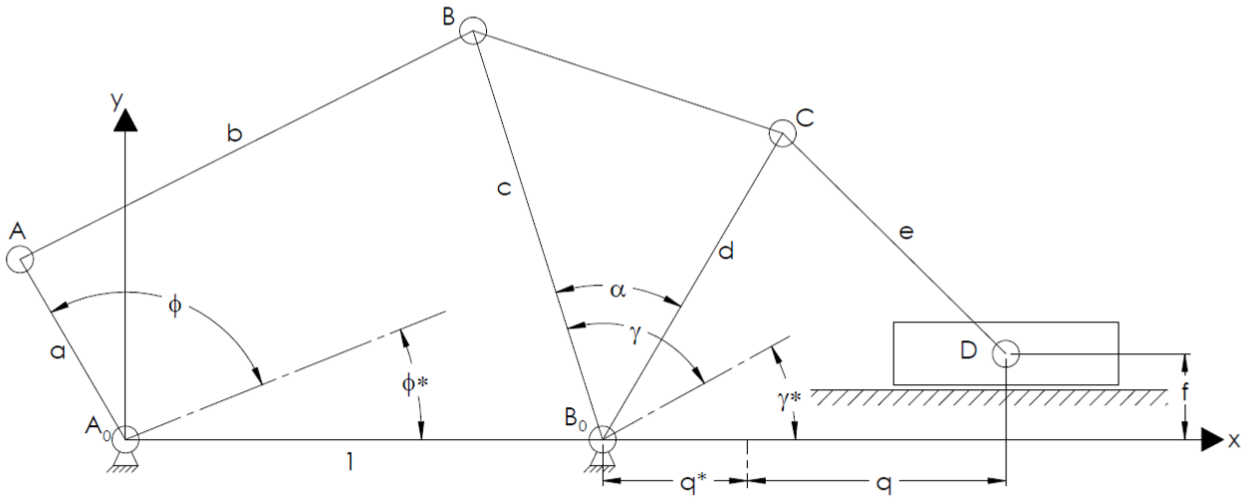

Figure 1. Kinematic diagram of a Watt II mechanism

The origin of the coordinate frame is at A_0 and the x-axis is along A_0B_0. The link lengths of the mechanism for design are $|A_0A| = a$, $|AB| = b$, $|B_0B| = c$, $\angle BB_0C = \alpha$, $|B_0C| = d$, $|CD| = e$ and the distance of point D to the x-axis is f. In Fig. 1, it is assumed that the slider displacement direction is along the x-axis, i.e. along A_0B_0. In general there might be a constant angle, say β, between the x-axis and the sliding direction of D. However, notice that the effect of the constant angles α and β to the I/O equation of the slider-crank loop is cumulative, therefore without loss of generality one may assume $\beta = 0$ as in Fig. 1. If the designer wishes to have a nonzero β after the synthesis computations are performed, it is possible to select and arbitrary angle β and modify angle $\angle BB_0C$ as $\alpha - \beta$ instead of α.

The input of the mechanism is angle ϕ and the output is the distance q. Angle γ is an intermediate variable to be used as the output of the four-bar loop and at the same time, the input of the slider-crank loop. In general the input angle can be measured from an inclined reference axis which makes an angle ϕ^* with the x-axis. ϕ^* can be used as a design parameters along with the link lengths. Similarly, angle γ may be measured from a reference axis which makes an angle γ^* with the x-axis. Also, the distance q may be measured from a constant distance q^* measured from B_0, which can be used as a design parameter.

$y = f(x)$ is to be generated for $x_0 \leq x \leq x_f$ using the six-link mechanism. The function $y = f(x)$ is decomposed into two as $w = g(x)$ and $y = h(w) = h(g(x)) = f(x)$. The intermediate function $g(\cdot)$ can be selected arbitrarily. Initial and final values of w and y are computed as $w_0 = g(x_0)$, $w_f = g(x_f)$, $y_0 = f(x_0)$ and $y_f = f(x_f)$. Let $\Delta x = x_f - x_0$, $\Delta w = w_f - w_0$ and $\Delta y = y_f - y_0$. The function variables x, w, y are related with the mechanism variables ϕ, γ, q linearly as follows:

$$\frac{\phi-\phi_0}{\Delta\phi}=\frac{x-x_0}{\Delta x}\ ,\ \frac{\gamma-\gamma_0}{\Delta\gamma}=\frac{w-w_0}{\Delta w}\ ,\ \frac{q-q_0}{\Delta q}=\frac{y-y_0}{\Delta y} \tag{1}$$

where $\phi_0 \le \phi \le \phi_f$, $\gamma_0 \le \gamma \le \gamma_f$, $q_0 \le q \le q_f$ and $\Delta\phi = \phi_f - \phi_0$, $\Delta\gamma = \gamma_f - \gamma_0$, $\Delta q = q_f - q_0$. The limits of the mechanism variables can be chosen arbitrarily. For given desired values of the function variables x, w, y, the corresponding mechanism variables ϕ, γ, q can be determined from Eq. (1) as:

$$\phi=\frac{\Delta\phi}{\Delta x}\left(x-x_0\right)+\phi_0\ ,\ \gamma=\frac{\Delta\gamma}{\Delta w}\left(w-w_0\right)+\gamma_0\ ,\ \psi=\frac{\Delta\psi}{\Delta y}\left(y-y_0\right)+\psi_0 \tag{2}$$

Eq. (2) is used for determining the precision points for the interpolation approximation method. Conversely, x, w, y can be determined in terms of ϕ, γ and q as.

$$x=\frac{\Delta x}{\Delta\phi}\left(\phi-\phi_0\right)+x_0\ ,\ w=\frac{\Delta w}{\Delta\gamma}\left(\gamma-\gamma_0\right)+w_0\ ,\ y=\frac{\Delta y}{\Delta q}\left(q-q_0\right)+y_0 \tag{3}$$

Eq. (3) is used after the synthesis procedure for checking the error between the desired y(x) and the generated y(q).

Formulation of the Design Equations and the Correction Method.

In (Kiper et al., 2017), three correction methods are presented for function generation with two-loop mechanisms. Correction method #1 assumes zero variable references f*, g*, q*, etc., whereas correction method #2 assumes nonzero variable references. The precision points (points of zero error) for both loops are chosen to be common in these two correction methods. In correction method #3, the synthesis procedure for the first loop is the same as the other methods; however instead of equating the precision points of the two loops, the points which correspond to the extrema of the error in the first loop are used for the second loop. It is possible to use all three correction methods for the mechanism in Fig. 1, but for brevity only one correction method is used in this paper. As explained in Section 1, only correction method #2 is used in this paper.

The I/O equation for the four-bar loop A_0ABB_0 reads

$$\left|\overrightarrow{AB}\right|=\left|\overrightarrow{A_oB}-\overrightarrow{A_oA}\right|\Rightarrow b^2=\left(1+cc\left(\gamma+\gamma^*\right)-ac\left(\phi+\phi^*\right)\right)^2+\left(cs\left(\gamma+\gamma^*\right)-as\left(\phi+\phi^*\right)\right)^2$$

$$\Rightarrow-\frac{1+a^2-b^2+c^2}{2cc\gamma^*}+\frac{ac\phi^*}{cc\gamma^*}c\phi-\frac{as\phi^*}{cc\gamma^*}s\phi+\frac{ac\left(\phi^*-\gamma^*\right)}{c\gamma^*}c\left(\gamma-\phi\right)+\frac{as\left(\phi^*-\gamma^*\right)}{c\gamma^*}s\left(\gamma-\phi\right)+t\gamma^*s\gamma=c\gamma$$

(4)

where c, s and t are short for cosine, sine and tangent, respectively. Eq. (4) can be written in polynomial form for five precision points as

$$\sum_{j=1}^{6}P_jf_j\left(\mathbf{x}_i\right)-F\left(\mathbf{x}_i\right)=0\ \text{ for i = 1, ..., 5} \tag{5}$$

where $\mathbf{x}_i = \{f_i, g_i\}$, $P_1 = -\frac{1+a^2-b^2+c^2}{2cc\gamma^*}$, $P_2 = \frac{ac\phi^*}{cc\gamma^*}$, $P_3 = \frac{as\phi^*}{cc\gamma^*}$, $P_4 = \frac{ac(\phi^*-\gamma^*)}{c\gamma^*}$, $P_5 = \frac{as(\phi^*-\gamma^*)}{c\gamma^*}$, $P_6 = t\gamma^*$, $f_1(\mathbf{x}_i)=1$, $f_2(\mathbf{x}_i)=c\phi_i$, $f_3(\mathbf{x}_i)=-s\phi_i$, $f_4(\mathbf{x}_i)=c(\gamma_i-\phi_i)$, $f_5(\mathbf{x}_i)=s(\gamma_i-\phi_i)$, $f_6(\mathbf{x}_i)=s\gamma_i$ and $F(\mathbf{x}_i)=c\gamma_i$. There are five design parameters (a, b, c, f* and g*) in Eq. (5), so there should be five precision points: $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\mathbf{x}_4$ and $\mathbf{x}_5$. However there are six P_j's, hence they cannot be independent of each other. Indeed, $P_4(P_3 - P_2P_6) = P_5(P_2 + P_3P_6)$. The problem can be linearized by using a Lagrange's variable l. Let P_6 = l and $P_j = m_j + n_j l$ for j = 1, 2, 3, 4, 5. Substituting into Eq. (5)

$$\begin{aligned}
m_1+n_1\lambda+(m_2+n_2\lambda)c\phi_1-(m_3+n_3\lambda)s\phi_1+(m_4+n_4\lambda)c(\gamma_1-\phi_1)+(m_5+n_5\lambda)s(\gamma_1-\phi_1)&=c\gamma_1-\lambda s\gamma_1\\
m_1+n_1\lambda+(m_2+n_2\lambda)c\phi_2-(m_3+n_3\lambda)s\phi_2+(m_4+n_4\lambda)c(\gamma_2-\phi_2)+(m_5+n_5\lambda)s(\gamma_2-\phi_2)&=c\gamma_2-\lambda s\gamma_2\\
m_1+n_1\lambda+(m_2+n_2\lambda)c\phi_3-(m_3+n_3\lambda)s\phi_3+(m_4+n_4\lambda)c(\gamma_3-\phi_3)+(m_5+n_5\lambda)s(\gamma_3-\phi_3)&=c\gamma_3-\lambda s\gamma_3\\
m_1+n_1\lambda+(m_2+n_2\lambda)c\phi_4-(m_3+n_3\lambda)s\phi_4+(m_4+n_4\lambda)c(\gamma_4-\phi_4)+(m_5+n_5\lambda)s(\gamma_4-\phi_4)&=c\gamma_4-\lambda s\gamma_4\\
m_1+n_1\lambda+(m_2+n_2\lambda)c\phi_5-(m_3+n_3\lambda)s\phi_5+(m_4+n_4\lambda)c(\gamma_5-\phi_5)+(m_5+n_5\lambda)s(\gamma_5-\phi_5)&=c\gamma_5-\lambda s\gamma_5
\end{aligned} \tag{6}$$

In order for Eqs. (6) to be satisfied for an arbitrary λ, the coefficients of λ and the rest of each equation should be equal to zero. In matrix form:

$$\begin{bmatrix}1 & c\phi_1 & -s\phi_1 & c(\gamma_1-\phi_1) & s(\gamma_1-\phi_1)\\ 1 & c\phi_2 & -s\phi_2 & c(\gamma_2-\phi_2) & s(\gamma_2-\phi_2)\\ 1 & c\phi_3 & -s\phi_3 & c(\gamma_3-\phi_3) & s(\gamma_3-\phi_3)\\ 1 & c\phi_4 & -s\phi_4 & c(\gamma_4-\phi_4) & s(\gamma_4-\phi_4)\\ 1 & c\phi_5 & -s\phi_5 & c(\gamma_5-\phi_5) & s(\gamma_5-\phi_5)\end{bmatrix}\begin{bmatrix}m_1\\m_2\\m_3\\m_4\\m_5\end{bmatrix}=\begin{bmatrix}c\gamma_1\\c\gamma_2\\c\gamma_3\\c\gamma_4\\c\gamma_5\end{bmatrix} \text{ and } \begin{bmatrix}1 & c\phi_1 & -s\phi_1 & c(\gamma_1-\phi_1) & s(\gamma_1-\phi_1)\\ 1 & c\phi_2 & -s\phi_2 & c(\gamma_2-\phi_2) & s(\gamma_2-\phi_2)\\ 1 & c\phi_3 & -s\phi_3 & c(\gamma_3-\phi_3) & s(\gamma_3-\phi_3)\\ 1 & c\phi_4 & -s\phi_4 & c(\gamma_4-\phi_4) & s(\gamma_4-\phi_4)\\ 1 & c\phi_5 & -s\phi_5 & c(\gamma_5-\phi_5) & s(\gamma_5-\phi_5)\end{bmatrix}\begin{bmatrix}n_1\\n_2\\n_3\\n_4\\n_5\end{bmatrix}=\begin{bmatrix}-s\gamma_1\\-s\gamma_2\\-s\gamma_3\\-s\gamma_4\\-s\gamma_5\end{bmatrix} \tag{7}$$

m_1, m_2, m_3, m_4, m_5, n_1, n_2, n_3, n_4 and n_5 are solved from Eqs.
Ошибка! Источник ссылки не найден. by matrix inversion. λ is solved from $P_4(P_3 - P_2P_6) = P_5(P_2 + P_3P_6)$:

$$P_4(P_3-P_2P_6)-P_5(P_2+P_3P_6)=(m_4+n_4\lambda)\left[m_3+n_3\lambda-\lambda(m_2+n_2\lambda)\right]-(m_5+n_5\lambda)\left[m_2+n_2\lambda+\lambda(m_3+n_3\lambda)\right]=0$$

$$\Rightarrow\left\{\begin{array}{l}(n_3n_5+n_2n_4)\lambda^3+\left[m_5n_3+m_4n_2+n_5(m_3+n_2)+n_4(m_2-n_3)\right]\lambda^2\\ +\left[m_5(m_3+n_2)+m_2n_5+m_4(m_2-n_3)-m_3n_4\right]\lambda+m_2m_5-m_3m_4\end{array}\right\}=0 \tag{8}$$

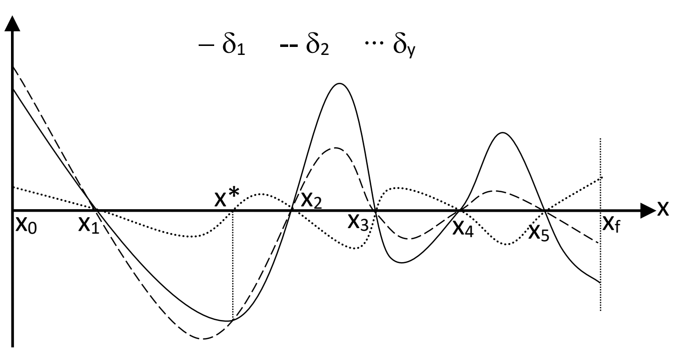

Figure 2. Error curves for the loops (δ_1 and δ_2) and function output (δ_y)

Eq. **Ошибка! Источник ссылки не найден.** is a cubic equation in λ and can be solved analytically. Either there are one real and two imaginary solutions or three real solutions. In case of multiple solutions either solution can be used. Then, $P_j = m_j + n_j\lambda$ are determined for j = 1, 2, 3, 4, 5.

Finally, the design parameters are computed as $\gamma^* = \tan^{-1} P_6$, $\phi^*=\text{atan2}(P_2, P_3)$, $a=\dfrac{P_4 c\gamma^*}{c(\phi^*-\gamma^*)}$, $c=\dfrac{ac\phi^*}{P_2 c\gamma^*}$ and $b=\sqrt{1+a^2+c^2+2cc\gamma^* P_1}$. $\gamma^* = \tan^{-1} P_6 + \pi$ is also possible. Once γ^* value is selected, ϕ^*, a, c and b are uniquely determined in terms of the P_j's, provided that b is real. a or c may be negative, in which case the limits of ϕ or γ should be increased by π. The resulting error variation is zero, that is $\delta_1 = w_{desired} - w_{generated1} = 0$, at least at five precision points (x_1, x_2, x_3, x_4 and x_5) if there is no branching problem, i.e. if $\delta_1 = 0$ at all precision points in the same assembly mode of the loop. The variation of the error curve δ_1 with respect to the function input x looks like the curve in Fig. 2.

In order to be able to compare the errors due to both loops, we assume that the outputs of both the four-bar and the slider-crank loops are link BB_0C and the output variable is γ. Hence we assume that slider displacement q is the input of the slider-crank loop and hence q is known as a linear function of the desired y values. The resulting γ as the output of the loop, and hence w values are obtained from the I/O equation of the slider-crank loop. Let $\delta_2 = w_{desired} - w_{generated2}$ as the error for given $y_{desired}(x)$ and the corresponding linearly related q values. For the dimensional synthesis of the slider-crank loop, the same precision points as the four-bar loop are used and δ_2 and δ_1 are forced to be approximately equal by changing the free variables such as the angle limits ϕ_0, ϕ_f, γ_0, γ_f. Changing linear variable limits q_0, q_f only affects the scale of the slider-crank loop, but has not effect on the amount of the generation error. The I/O equation for the slider-crank loop is given by

$$\begin{aligned} &\left|\overrightarrow{CD}\right| = \left|\overrightarrow{B_oD} - \overrightarrow{B_oC}\right| \Rightarrow e^2 = \left[q^* + q - dc(\gamma+\gamma^*-\alpha)\right]^2 + \left[ds(\gamma+\gamma^*-\alpha) - f\right]^2 \\ &\Rightarrow q^{*2} + d^2 - e^2 + f^2 + 2q^*q + q^2 - 2d(q^*+q)c(\gamma+\gamma^*-\alpha) - 2dfs(\gamma+\gamma^*-\alpha) = 0 \\ &\Rightarrow \left\{ \begin{array}{c} \dfrac{q^{*2}+d^2-e^2+f^2}{2dc(\gamma^*-\alpha)} + \dfrac{q^*}{dc(\gamma^*-\alpha)} q + \dfrac{1}{2dc(\gamma^*-\alpha)} q^2 + t(\gamma^*-\alpha) q s\gamma \\ + \left[q^* t(\gamma^*-\alpha) - f\right] s\gamma - \left[q^* + ft(\gamma^*-\alpha)\right] c\gamma \end{array} \right\} = qc\gamma \end{aligned} \tag{7}$$

Eq. (7) can be written in polynomial form Eq. (5) for five precision points where $\mathbf{x}_i = \{\gamma_i, q_i\}$, $P_1 = \dfrac{q^{*2}+d^2-e^2+f^2}{2dc(\gamma^*-\alpha)}$, $P_2 = \dfrac{q^*}{dc(\gamma^*-\alpha)}$, $P_3 = \dfrac{1}{2dc(\gamma^*-\alpha)}$, $P_4 = t(\gamma^*-\alpha)$, $P_5 = q^* t(\gamma^*-\alpha) - f$, $P_6 = q^* + ft(\gamma^*-\alpha)$, $f_1(\mathbf{x}_i) = 1$, $f_2(\mathbf{x}_i) = q_i$, $f_3(\mathbf{x}_i) = q_i^2$, $f_4(\mathbf{x}_i) = q_i s\gamma_i$, $f_5(\mathbf{x}_i) = s\gamma_i$, $f_6(\mathbf{x}_i) = -c\gamma_i$ and $F(\mathbf{x}_i) = q_i c\gamma_i$. The five precision points are selected as a function of y_i, hence as a function of x_i for i = 1, …, 5, where x_i are the precision points used for the four-bar loop. There are six P_j's in terms of five design parameters: α, d, e, f and q*. P_j's are interrelated as $P_2(1+P_4^2) - 2P_3(P_4P_5 + P_6) = 0$. Let $P_4 = \lambda$ and $P_j = m_j + n_j\lambda$ for j = 1, 2, 3, 5, 6. Substituting into Eq. (7):

$$
\begin{aligned}
&m_1+n_1\lambda+(m_2+n_2\lambda)q_1+(m_3+n_3\lambda)q_1^2+(m_5+n_5\lambda)s\gamma_1-(m_6+n_6\lambda)c\gamma_1=q_1c\gamma_1-\lambda q_1s\gamma_1\\
&m_1+n_1\lambda+(m_2+n_2\lambda)q_2+(m_3+n_3\lambda)q_2^2+(m_5+n_5\lambda)s\gamma_2-(m_6+n_6\lambda)c\gamma_2=q_2c\gamma_2-\lambda q_2s\gamma_2\\
&m_1+n_1\lambda+(m_2+n_2\lambda)q_3+(m_3+n_3\lambda)q_3^2+(m_5+n_5\lambda)s\gamma_3-(m_6+n_6\lambda)c\gamma_3=q_3c\gamma_3-\lambda q_3s\gamma_3\\
&m_1+n_1\lambda+(m_2+n_2\lambda)q_4+(m_3+n_3\lambda)q_4^2+(m_5+n_5\lambda)s\gamma_4-(m_6+n_6\lambda)c\gamma_4=q_4c\gamma_4-\lambda q_4s\gamma_4\\
&m_1+n_1\lambda+(m_2+n_2\lambda)q_5+(m_3+n_3\lambda)q_5^2+(m_5+n_5\lambda)s\gamma_5-(m_6+n_6\lambda)c\gamma_5=q_5c\gamma_5-\lambda q_5s\gamma_5
\end{aligned}
\tag{8}
$$

Separating the coefficients of λ and the rest of each equation in Eqs. (8) and writing in mat-rix form:

$$
\begin{bmatrix}1&q_1&q_1^2&s\gamma_1&-c\gamma_1\\1&q_2&q_2^2&s\gamma_2&-c\gamma_2\\1&q_3&q_3^2&s\gamma_3&-c\gamma_3\\1&q_4&q_4^2&s\gamma_4&-c\gamma_4\\1&q_5&q_5^2&s\gamma_5&-c\gamma_5\end{bmatrix}\begin{bmatrix}m_1\\m_2\\m_3\\m_5\\m_6\end{bmatrix}=\begin{bmatrix}q_1c\gamma_1\\q_2c\gamma_2\\q_3c\gamma_3\\q_4c\gamma_4\\q_5c\gamma_5\end{bmatrix}\text{ and }\begin{bmatrix}1&q_1&q_1^2&s\gamma_1&-c\gamma_1\\1&q_2&q_2^2&s\gamma_2&-c\gamma_2\\1&q_3&q_3^2&s\gamma_3&-c\gamma_3\\1&q_4&q_4^2&s\gamma_4&-c\gamma_4\\1&q_5&q_5^2&s\gamma_5&-c\gamma_5\end{bmatrix}\begin{bmatrix}n_1\\n_2\\n_3\\n_5\\n_6\end{bmatrix}=\begin{bmatrix}-q_1s\gamma_1\\-q_2s\gamma_2\\-q_3s\gamma_3\\-q_4s\gamma_4\\-q_5s\gamma_5\end{bmatrix}
\tag{9}
$$

After m_1, m_2, m_4, m_5, m_6, n_1, n_2, n_4, n_5 and n_6 are solved from Eqs. (9) by matrix inversion, λ is determined using $P_2\left(1+P_4^2\right)-2P_3\left(P_4P_5+P_6\right)=0$:

$$
\begin{aligned}
&(m_2+n_2\lambda)(1+\lambda^2)-2(m_3+n_3\lambda)(\lambda(m_5+n_5\lambda)+m_6+n_6\lambda)=0\\
&\Rightarrow(n_2-2n_3n_5)\lambda^3+\left[m_2-2(m_3n_5+n_3(m_5+n_6))\right]\lambda^2+\left[n_2-2(n_3m_6+m_3(m_5+n_6))\right]\lambda+m_2-2m_3m_6=0
\end{aligned}
\tag{10}
$$

Eq. (10) can be analytically solved and results in either one or three real solutions for λ. Once λ is determined or selected, $P_3 = \lambda$ and $P_j = m_j + n_j\lambda$ are determined for j = 1, 2, 4, 5, 6. Finally the design parameters are solved as $\alpha=\gamma^*-\tan^{-1}P_4$ or $\alpha=\gamma^*-\tan^{-1}P_4+\pi$, $d=1/\left(2c(\gamma^*-\alpha)P_3\right)$, $q^*=P_2/(2P_3)$, $f=q^*P_4-P_5$ and $e=\sqrt{q^{*2}+d^2+f^2-P_1/P_3}$. α is selected so that d is positive. e, f and q* are determined uniquely provided that e is real.

Representative δ_1 and δ_2 variations versus the function input x are illustrated in Fig. 2. As a result of the whole design process, the q output values of the 6-link mechanism result in corresponding $y_{generated}$ values as the output of the generated function. For given function input x, and hence corresponding mechanism input angle ϕ, the error variance $\delta_y = y_{desired} - y_{generated}$ is also depicted in Fig. 2. Definitely $\delta_y = 0$ at the precision points x_1, x_2 and x_3. There may be other points where $\delta_y = 0$ whenever δ_1 curve intersects δ_2, such as x* in Fig. 2.

The closer δ_1 and δ_2 curves, the lower are the δ_y values. In order to obtain lower δ_y error values the designer can adjust several freely selected parameters such as the limits ϕ_0, ϕ_f, γ_0, γ_f of the input joint variable ϕ and intermediate joint variable γ of the mechanism. Also, it is possible to adjust the intermediate function g(·) for most of the functions. When a software such as Microsoft Excel® is used for the computations, the designer can make use of spin buttons for varying the limits of the ϕ, γ and q and, if available, the intermediate function parameter(s) for g(·). By continuously changing the free parameters, the designer can immediately see the tendency of change in the error variations

δ_1, δ_2 and δ_y. At the same time, it is possible to monitor a proper norm of the error, such as the maximum error $|\delta_y|_{max}$ or rms error of δ_y and minimize it. Meanwhile, certain design considerations such as maximum link length to minimum link length ratio, transmission angles, etc. can be monitored.

Numerical Example

The formulations in Section 3 are implemented in Excel and a design environment is constructed which can be used for any arbitrary function. For the example in Fig. 3, the function to be generated is $y = x^2$ for $1 \le x \le 5$. The intermediate function is $g(x) = x^k$, where k is an adjustment parameter for the designer. The synthesis computations described in Section 3 are implemented in the Loop1 and Loop2 sheets. In the sheet shown in Fig. 3 the designer can adjust the joint variable limits ϕ_0, ϕ_f, γ_0, γ_f, q_0, q_f with spin buttons; the configuration of the loops (config1 and config2); and select the Lagrange variable values λ_1 and λ_2 for the two loops – each out of three possible solutions from their respective cubic equations. By these adjustments, error variation curves δ_1, δ_2 and δ_y are monitored simultaneously. Also the maximum error $|\delta_y|_{max}$ and the ratio of the longest link (better less than 10) to the shortest link is monitored. Also the joint variable ranges $\Delta\phi$ and $\Delta\gamma$ should not be too small (better more than 20°). Also the mechanism is drawn and its motion can be simulated with a spin button. A good result is obtained usually in less than in hour – in less time than running any numerical optimization algorithm.

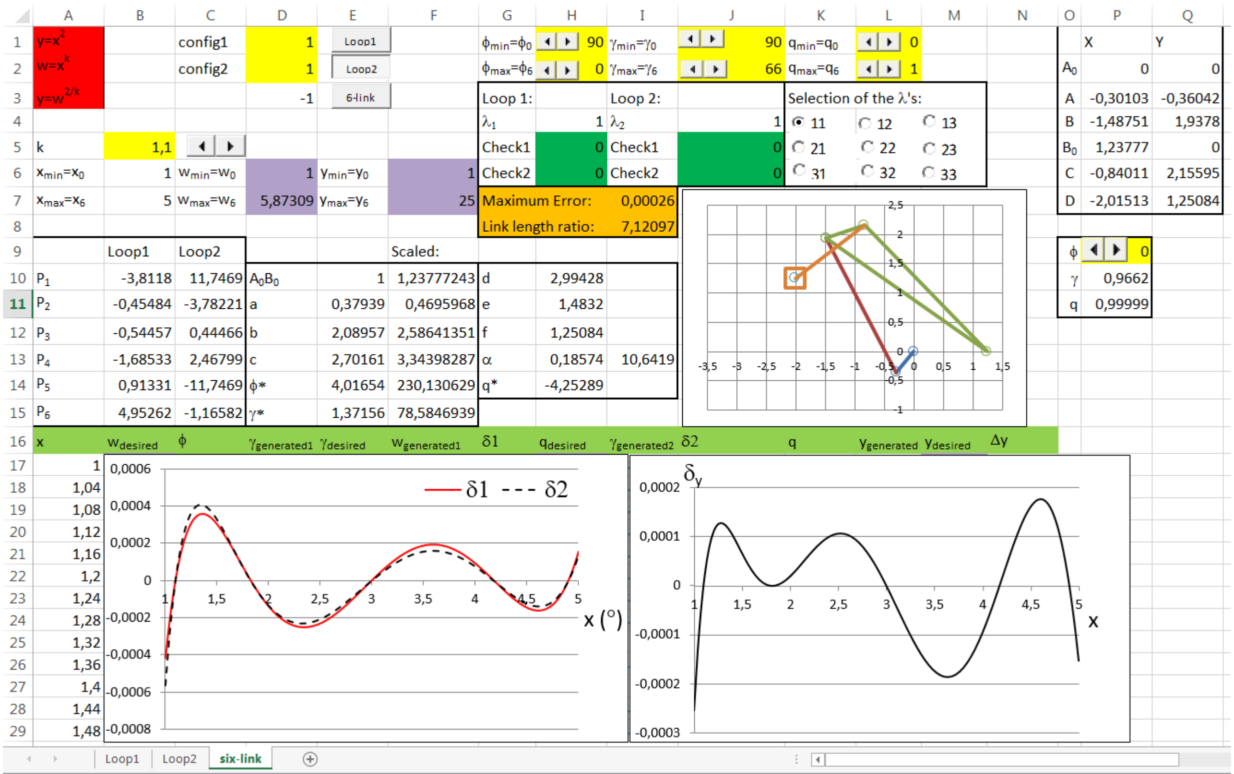

Figure 3. Excel design sheet

After several trials, a good result for the maximum error $|\delta_y|_{max} = 2.6 \times 10^{-4}$ is obtained for $|A_0B_0| = 1$, $|A_0A| = a = 0.379$, $|AB| = b = 2.090$, $|B_0B| = c = 2.702$, $\phi^* = 230.1°$, $\gamma^* = 78.6°$, $|B_0C| = d = 2.994$, $|CD| = e = 1.483$, $D_y = f = 1.251$, $\alpha = 10.6°$ and $q^* = -4.253$. The link lengths 1, a, b, c of the four-bar loop can be scaled arbitrarily. It is observed during the computations that the slider variable q has no effect on the error variations. The designer can adjust $\Delta q = q_f - q_0$ in order to scale the slider-crank loop link lengths d, e, f. The slider direction can also be adjusted by modifying the angle α as described in Section 2.

Conclusions

In this paper, the method of decomposition and correction is applied for a Watt II type planar six-link mechanism with prismatic output. An analytical method for determining five design parameters for each loop, hence a total number of ten design parameters is presented. There are several free design parameters, such as the limits of the input and intermediate angles of the mechanism and the parameter or parameters that appear during the decomposition of the function to be generated. Also there may be multiple solutions due to the solution of the nonlinear equation in terms of Lagrange parameters. These free design parameters and options for the Lagrange parameters gives a great amount of flexibility to the designer in order to minimize the generation error while considering several constraints such as link lengths ratios and ranges of the joint variables. The method presented in the paper is illustrated with a numerical example. $y = x^2$ is generated for $1 \leq x \leq 5$ with a maximum error value of 2.6×10^{-4} for y. The generation precision is very good when compared to the other results in the literature.

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